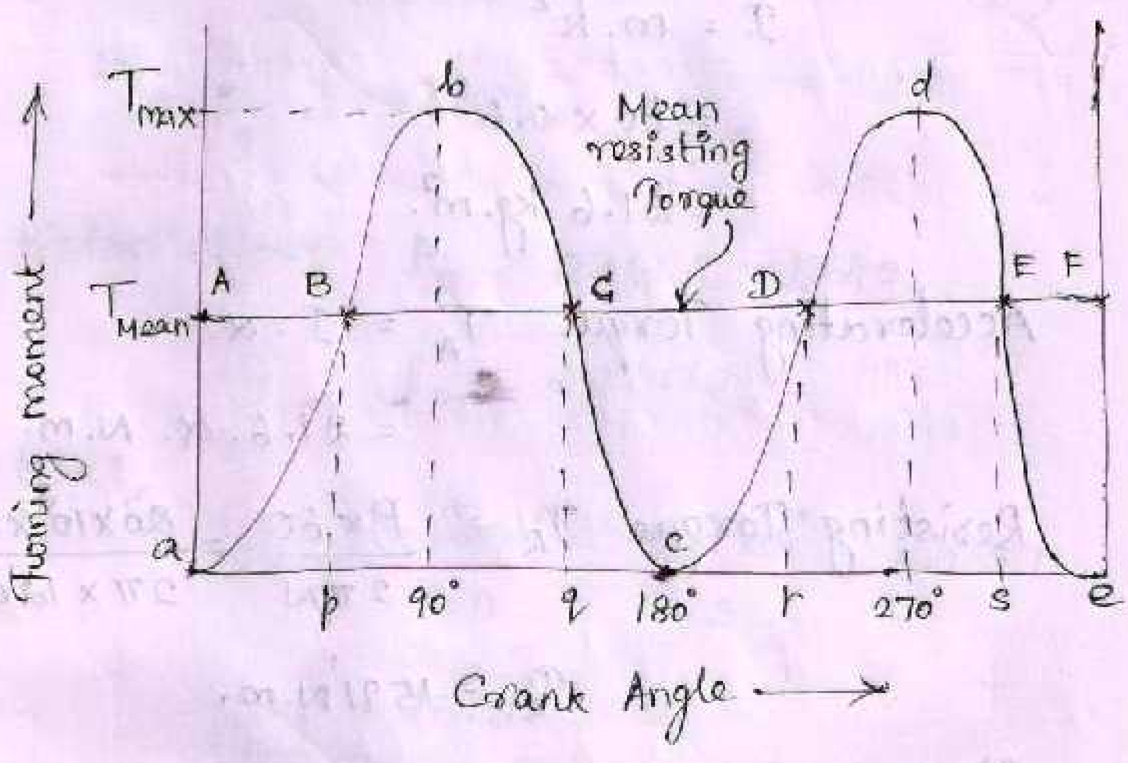
Turning moment diagram

The turning moment diagram (also known as crank effort diagram) is the graphical representation of the turning moment or crank effort for various positions of the crank.

Turning moment diagram for a single cylinder double acting steam engine.



Turning moment on the crank shaft

$$T = F_p \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

where,

F_p = Piston effort

r = radius of crank

n = Ratio of the connecting rod length and radius of crank

θ = Angle turned by the crank from TDC.

From the above expression.

$T = 0$; when crank angle $\theta = 0$ & 180°

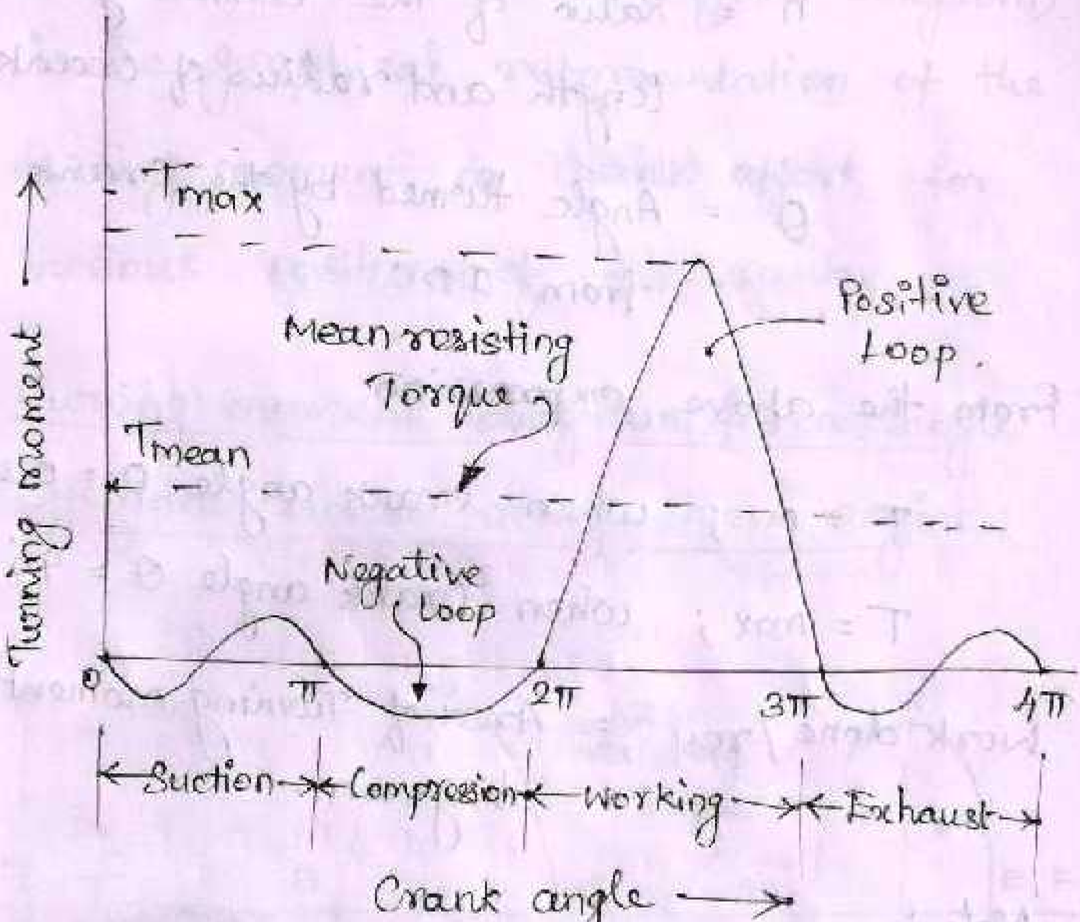
$T = max$; when crank angle $\theta = 90^\circ$.

Work done / rev = Area of Turning moment diagram.

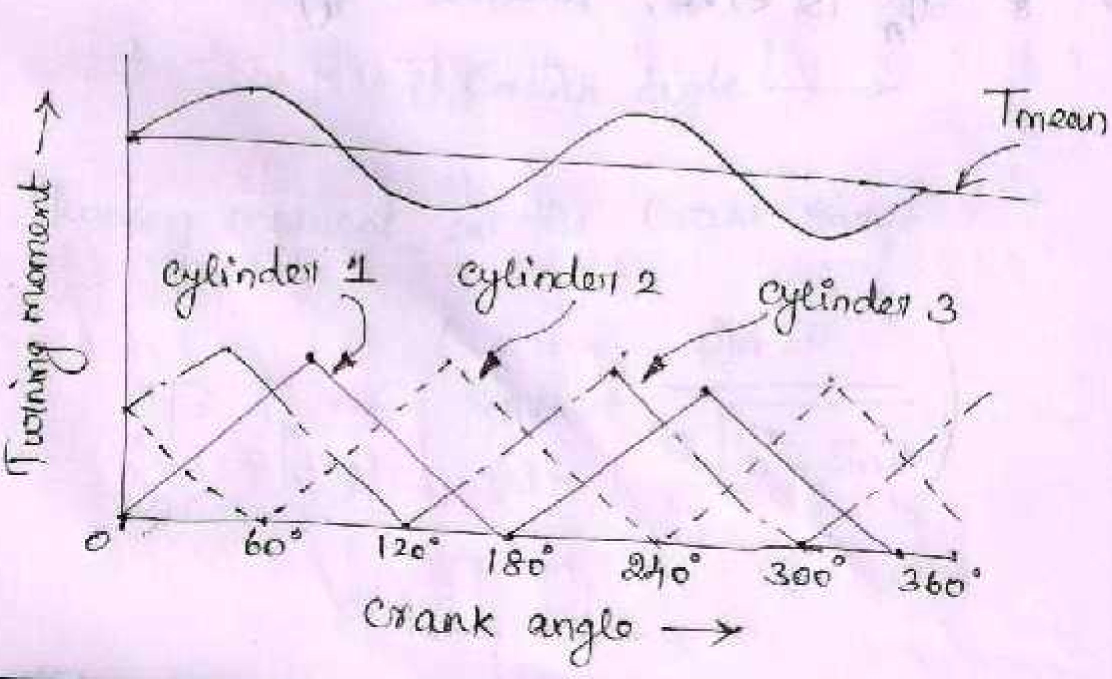
Note :-

1. Accelerating Torque $T_A = T - T_{mean}$.
2. T_A is (+)ve ; when the flywheel Accelerates.
3. T_A is (-)ve ; when the flywheel retards.

Turning moment diagram for a Four Stroke Cycle I.C. Engine



Turning moment diagram for a Multi cylinder engine.



Fluctuation of Energy.

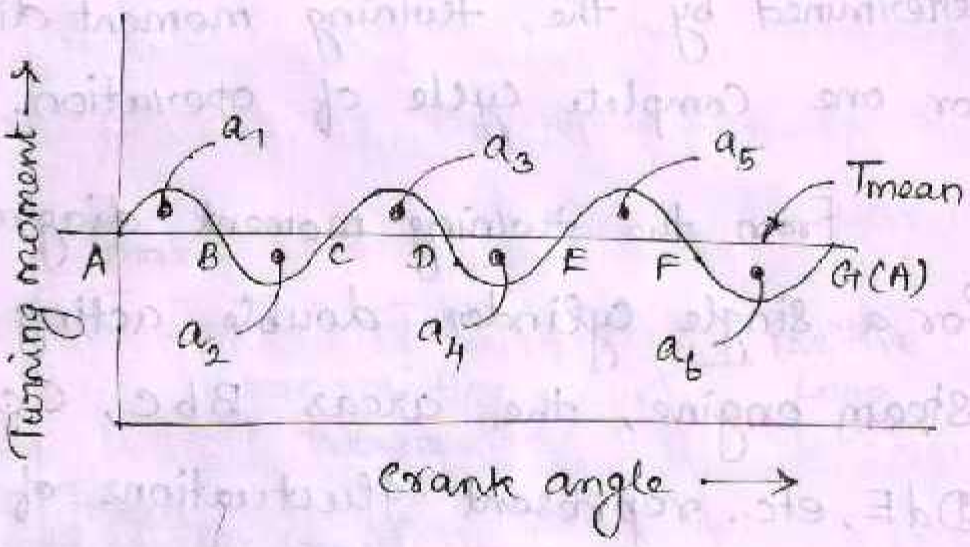
The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation.

From the turning moment diagram for a single cylinder double acting steam engine, the areas BbC, CcD, DdE, etc. represent fluctuations of energy.

The variations of energy above and below the mean resisting torque line are called fluctuations of Energy.

The difference between the maximum and the minimum energies is known as maximum fluctuation of energy.

Determination of maximum fluctuation of Energy.



The turning moment diagram for a multi cylinder engine is shown by a wavy curve in fig. There areas a_1 a_2 a_3 a_4 a_5 a_6 represents some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

Let the energy in the flywheel at A.

at A $= E$

Energy at B $= E + a_1$

C $= E + a_1 - a_2$

D $= E + a_1 - a_2 + a_3$

E $= E + a_1 - a_2 + a_3 - a_4$

F $= E + a_1 - a_2 + a_3 - a_4 + a_5$

G $= E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

Energy at A = Energy at G.

maximum energy in the flywheel.

$$= E + a_1$$

minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4.$$

Max. fluctuation of energy

$$\Delta E = \text{Max. Energy} - \text{Min. Energy}$$

$$\Delta E = a_2 - a_3 + a_4.$$

Coefficient of Fluctuation of Energy.

It is defined as ratio of the maximum fluctuation of energy to the work done per cycle.

$$C_E = \frac{\text{Max. fluctuation of Energy}}{\text{Work done / cycle}}$$

Note:

1. Work done / cycle = $T_{mean} \times \theta$.

$\theta = 2\pi$ (2 stroke I.C. engine)

$\theta = 4\pi$ (4 stroke I.C. engine).

2. Work done / cycle = $\frac{P \times 60}{n}$.

$n = N$. (2 stroke)

$n = N/2$ (4 stroke).

Flywheel :-

A Flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

A flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.

Coefficient of fluctuation of speed.

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed.

The ratio of the maximum fluctuation of speed to the mean speed is called Coeff. of Fluct. of speed.

$$C_s = \frac{N_1 - N_2}{N} = \frac{\omega_1 - \omega_2}{\omega}.$$

N_1 & N_2 = max. and min. speeds in rpm.

$$N = \text{mean speed} = \frac{N_1 + N_2}{2}.$$

C_s is a limiting factor in the design of flywheel.

<u>Coefficient of steadiness (m).</u>

$$m = \frac{1}{C_s} = \frac{N}{N_1 - N_2}.$$

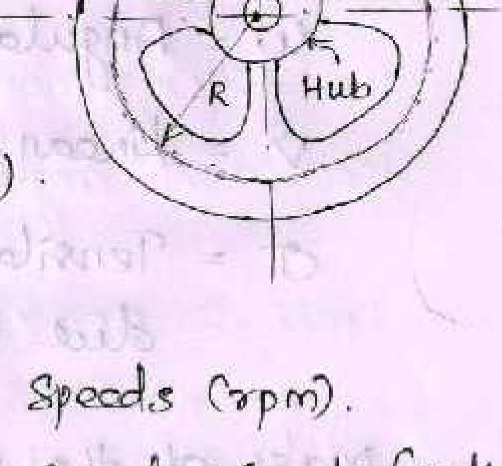
<u>Energy stored in flywheel.</u>

m = mass of flywheel (kg)

k = Radius of gyration of the flywheel (m)

I = moment of inertia of flywheel. ($kg.m^2$).

$= mk^2$.



N_1 & N_2 = max. and min. speeds (rpm).

ω_1 & ω_2 = max. and min. Angular speeds (rad/s).

N = mean speed

ω = mean angular speed.

C_s = Coefficient of fluctuation of speed.

Mean kinetic energy of the flywheel,

$$E = \frac{1}{2} I . \omega^2$$

$$= \frac{1}{2} m . k^2 . \omega^2 .$$

Maximum fluctuation of energy

$$\Delta E = \text{max } k.E - \text{min. } k.E.$$

$$\Delta E = m . k^2 \omega^2 . C_s .$$

<u>Dimensions of flywheel Rim.</u>

D = Mean diameter of rim.

A = Cross sectional area of rim

ρ = Density of rim material.

N = Speed of the flywheel.

ω = Angular velocity.

v = Linear velocity $= \omega . R$.

σ = Tensile stress or Hoop stress. due to centrifugal force

Mass of the rim = volume x Density

$$m = \pi . D . A . \rho .$$

$$\boxed{A = \frac{m}{\pi . D . \rho}}$$

Problem ① A multi cylinder engine is to run at a speed of 600 rpm. on drawing the turning moment diagram to a scale of 1 mm = 250 N.m and 1mm = 3°, the areas above and below the mean Torque line in mm^2. are : +160, −172, +168, −191, +197, −162. The speed is to be kept within ±1% of the mean speed of the engine. Calculate the necessary moment of Inertia of the Flywheel. Determine the suitable dimensions of a rectangular flywheel rim if the breath is twice its thickness. The density of the cast iron is 7250 kg/m^3 and its hoop stress is 6 Mpa. Assume that the rim contributes 92% of the flywheel effect.

Given data:-

$N = 600$ rpm.

$\rho = 7250$ kgf/m^3

$\sigma = 6$ Mpa.

Now, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.84$ rad/s.

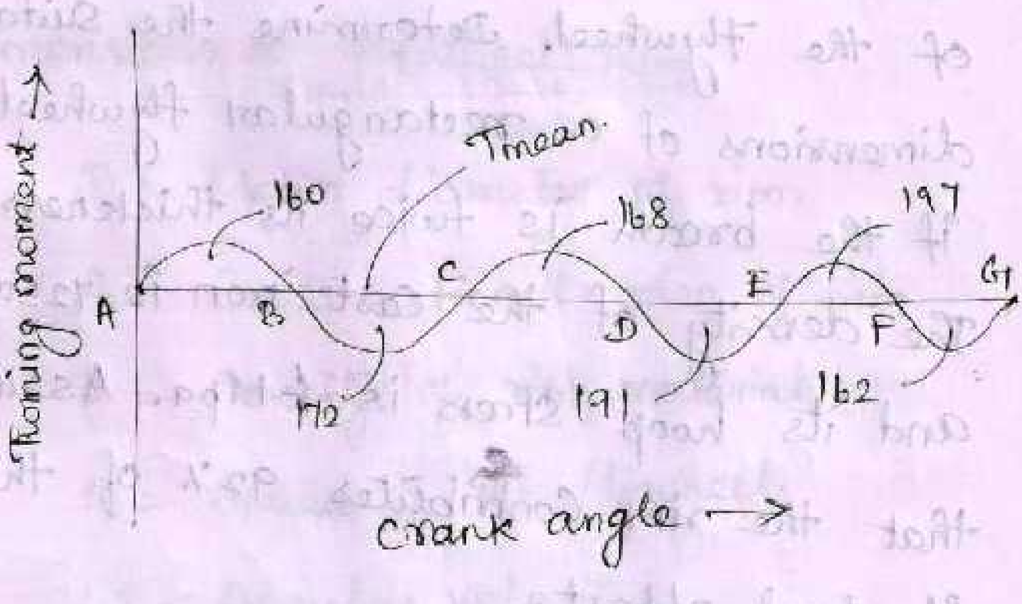
Since,

the fluctuation of speed is $\pm 1\%$ of mean speed.

$$\omega_1 - \omega_2 = 2\% \, \omega.$$
$$= 0.02 \, \omega$$

$$\Rightarrow C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02$$

moment of Inertia of the flywheel.



The turning moment scale is $1mm = 250 Nm$.

the crank angle scale is $1mm = 3^\circ = \frac{\pi}{60}$ rad.

$$1 mm^2 = 250 \times \frac{\pi}{60} = 13.1 N.m$$

Total energy at $A = E$

$$B = E + 160$$
$$C = E + 160 - 172$$
$$= E - 12$$

$D = E - 12 + 168 = E + 156$

$E = E + 156 - 191 = E - 35$

$F = E - 35 + 197 = E + 162$

$G = E + 162 - 162 = E$

max. fluctuation of energy

ΔE = max. energy - min. Energy

$= (E + 162) - (E - 35)$

$= 197 \text{ mm}^2$

$\Delta E = 197 \times 13.1 = 2581 \text{ N.m.}$

WKT,

$\Delta E = I . \omega^2 . C_s .$

$2581 = I \times (62.84)^2 \times 0.02$

$I = 32.7 \text{ kg.m}^2.$

Dimensions of the flywheel rim.

Let t = Thickness of flywheel rim

b = Breath of flywheel rim

D = mean diameter of flywheel rim.

WKT

Hoop stress $\left\{ \begin{array}{l} \sigma = \rho . V^2 . \\ 6 \times 10^6 = 7250 . V^2 . \end{array} \right.$

$V = 28.8 \text{ m/s}.$

WKT,

$$V = \frac{\pi D N}{60}$$

$$28.8 = \frac{\pi \times D \times 600}{60}$$

$$D = 0.92 \ m/s.$$

max. fluctuation of energy of rim,

$$\Delta E_{rim} = 0.92 \ \Delta E.$$

$$= 0.92 \times 2581$$

$$= 2375 \ N.m.$$

WKT, $\Delta E_{rim} = m . V^2 . C_s.$

$$2375 = m . (28.8)^2 \times 0.02$$

mass. $m = 143 \ kg.$

mass of flywheel = volume x Density

$$m = \pi . D . A . \rho$$

$$m = \pi . D . b . t . \rho$$

$$143 = \pi \times 0.92 \times 2t \times t \times 7250$$

$$t = 0.0584 \ m$$

$$t = 58.4 \ mm.$$

$$\Rightarrow b = 2t$$

$$b = 116.8 \ mm.$$

Problem ② An otto cycle engine develops 50kw at 150rpm. with 75 explosions per minute. The change of speed from the commencement to the end of power stroke must not exceed 0.5% of mean on either side. Find the mean diameter of the flywheel and a suitable rim cross sections having width four times the depth so that the Hoop stress does not exceed 4Mpa. Assume that the flywheel stores 16/15 times the energy stored by the rim and the work done during power stroke is 1.40 times the work done during the cycle. Density of rim material is 7200 kg/m³.

Given data:

$P = 50 \text{ kW}$

$N = 150 \text{ rpm} \Rightarrow \omega = (2\pi \times 150/60) = 15.71 \text{ rad/s}$

$n = 75$

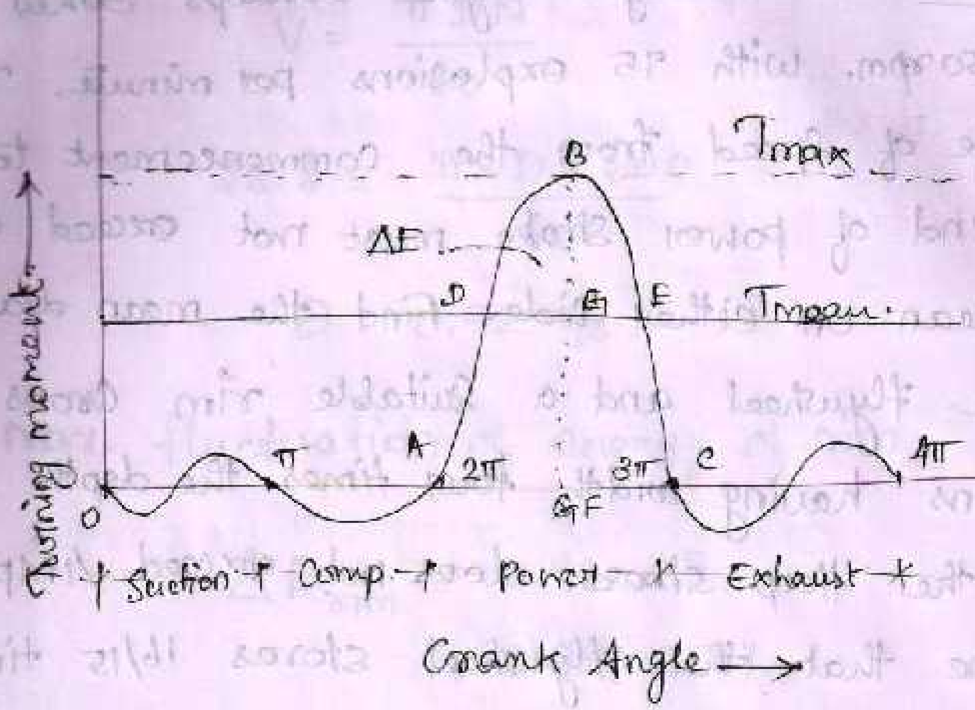
$\sigma = 4 \text{ Mpa}$

$\rho = 7200 \text{ kg/m}^3$

Power transmitted $P = T_{mean} \times \omega$

$$T_{mean} = \frac{50 \times 10^3}{15.71}$$

$$T_{mean} = 3182.7 \text{ N.m}$$



work done per cycle $= T_{mean} \times \theta$

$= 3182.7 \times 4\pi$

$= 40000$ N.m.

Work done during power stroke =

$= 1.4 \times$ work done per cycle

$= 1.4 \times 40000$

$= 56000$ N.m. ——— ①

From fig. ΔABC

Base AC = π radians

Height BF = T_{max}.

work done during working stroke.

$= \frac{1}{2} . \pi . T_{max}$

$= 1.571\ T_{max}$. ——— ②

From (1) & (2).

$$T_{max} = \frac{56000}{1.571}$$

$$T_{max} = 35646 \text{ N.m.}$$

WKT, the excess Torque

$$T_{Excess} = BG = BF - FG$$

$$= T_{max} - T_{mean}$$

$$= 35646 - 3182.7$$

$$= 32463.3 \text{ N.m.}$$

from llly ΔBDE & ΔABC.

$$\frac{DE}{AC} = \frac{BG}{BF}$$

$$DE = \frac{BG}{BF} \times AC$$

$$= \frac{32463.3}{35646} \times \pi$$

$$DE = 0.9107\pi.$$

Maximum fluctuation of energy,

$$\Delta E = \text{Area of } \Delta BDE$$

$$= \frac{1}{2} DE \times BG$$

$$= \frac{1}{2} \times 0.9107 \times 32463.3$$

$$\Delta E = 46445 \text{ N.m.}$$

Mean diameter of the flywheel

D = mean diameter of flywheel

v = Peripheral velocity of flywheel.

WKT, Hoop stress

$$\sigma = \rho v^2$$

$$4 \times 10^6 = 7200 \times v^2$$

$$v = 23.58 \text{ m/s}$$

WKT,

$$v = \frac{\pi D N}{60}$$

$$\Rightarrow D = \frac{23.58 \times 60}{\pi \times 150}$$

$$D = 3 \text{m}.$$

Cross Sectional dimensions of the rim.

Thickness of the rim = t.

width of the rim $b = 4t$.

Cross sectional area of rim =

$$A = b \times t.$$

$$A = 4t \times t.$$

mass of flywheel rim.

m = mass of flywheel.

E = Total energy of flywheel.

Since the fluctuation of speed is 0.5% of the mean speed on either side,

Total fluctuation of speed

$$N_2 - N_1 = 1\% \, N.$$

Coefficient of fluctuation of speed

$$C_s = \frac{N_1 - N_2}{N} = 0.01$$

max. fluctuation of energy ΔE

$$\Delta E = E \times 2 \times C_s = 0.02E.$$

$$E = \frac{46445}{0.02}$$

$$E = 2322 \times 10^3 \text{ N.m.}$$

Since, the energy stored by the flywheel is ~~1~~ 16/15 times the energy stored by the rim, therefore the energy of the rim.

$$E_{rim} = \frac{15}{16} E.$$

$$= \frac{15}{16} \times 2322 \times 10^3$$

$$E_{rim} = 2177 \times 10^3 \text{ N.m.}$$

WKT,

Energy of the rim $E_{rim} = \frac{1}{2} m v^2.$

$$2177 \times 10^3 = \frac{1}{2} \times m \times v^2$$

$$m = 7831 \text{ kg.}$$

WKT,

mass of the flywheel rim,

$$7831 = \pi D \times A \times \rho$$

$$7831 = \pi \times 3 \times 4t^2 \times 7200$$

$$t = 0.17 m = 170 mm$$

$$b = 680 mm.$$

Fly wheel in Punching Press.

The function of a flywheel in an engine is to reduce the fluctuations of speed, when the load on the crank shaft is constant and input Torque varies during the cycle.

The flywheel can also be used to perform the same function when the torque is constant and the load varies during the cycle. Such an application is found in Punching Press or in a rivetting machine

The crank is driven by a motor which supplies constant torque and the punch is at the position of the slider in a slider crank mechanism.

The load acts only during the rotation of the crank from θ_1 to θ_2.

Let E_1 = Energy required for a punching a hole.

d_1 = Dia. of hole punched

t_1 = Thickness of the plate.

τ_u = Ultimate shear stress for plate material.

Maximum shear force $F_s = \pi d_1 t_1 \tau_u$.

Work done $E_1 = \frac{1}{2} F_s t$

Energy supplied by motor to crank shaft during actual punching operation. $\Big\}$ $E_2 = E_1 \left(\frac{\theta_2 - \theta_1}{2\pi}\right)$.

Balance energy required for punching $\Big\}$ $= E_1 - E_2$.

This energy is to be supplied by the flywheel by the decrease in its kinetic energy when its speed falls from maximum to minimum. Thus

maximum fluctuation of energy ΔE.

$$\Delta E = E_1 - E_2$$

$$= E_1\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right).$$

The values of θ_1 and θ_2 may be determined only if the crank radius (r) length of connecting rod (l)

$$\frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}.$$

t = Thickness of material to be punched

s = stroke of the punch.

= 2 × crank radius

$s = 2r$.