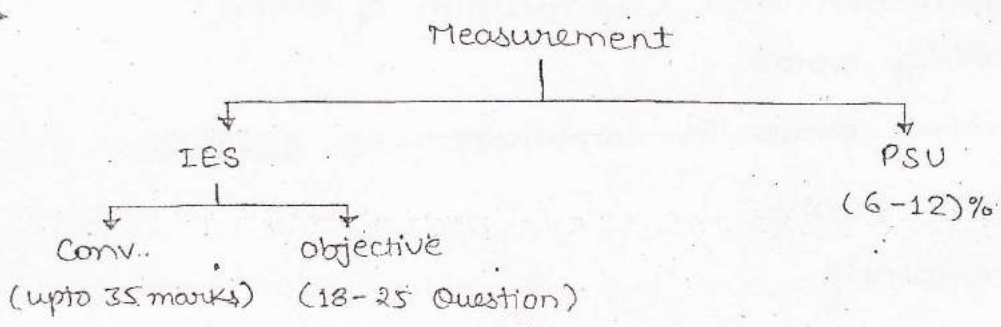


~: MEASUREMENT :~

⇒ Types of Question :-

- (i) Single Stand alone Statement (SSSQ) → (N) Numerical / (T) Theory
- (ii) Matching list type Question (MLQ) → Theory
- (iii) Combination of option (COOP) → Theory
- (iv) Assertion (A) and Reason (R)

⇒



⊙ Introduction to Measurement :-

- "Measurement is defined as a process of comparison b/w a standard and a unknown, resulting in knowing the magnitude of unknown in terms of the standard."
- "Instrument is a device that allow us to make this comparison."
- The two essential characteristic of any electrical instrument are :-
 - (a) The operational power consumption of the Instrument should be negligible.
 - (b) The instrument should not affect the ambient condⁿ of the ckt in which it have been introduced.

- The above two characteristics significantly increase the accuracy of Instrument where, "Accuracy is defined as the closeness with which the measured value approaches to true value."

⇒ Syllabus :-

(1) Error Analysis

- ↳ Introduction and classification of error.
- ↳ Limiting error.
- ↳ Limiting error in combination of quantities.
- ↳ Known errors.
- ↳ Uncertainty.

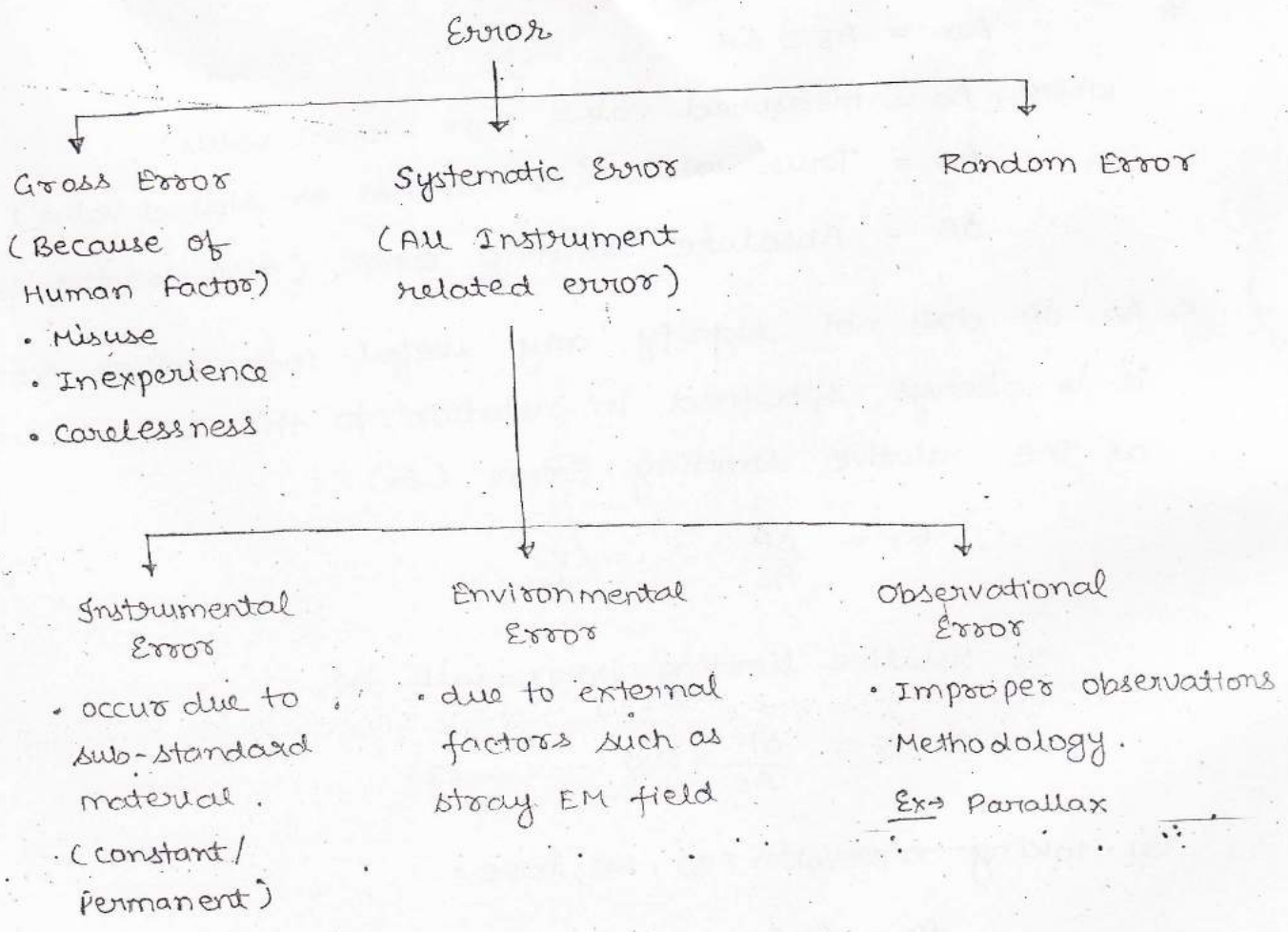
⊙ Error Analysis :-

⇒ Introduction and classification of error :-

- Accuracy of an instrument or component is always specified in terms of its error where, error is defined as the closeness deviation of the measured value from true value.
- Errors are broadly classified on the basis of their source, mode of propagation, probability of occurrence and the magnitude as,

Ex: $32 \pm 2\%$

Examples show that, value of the quantity is 32 with the accuracy of 98% ($100 - 2 = 98$) it means it has max. error of $\pm 2\%$ and its accuracy is 98%.



* Random Error :-

- Random errors are those whose source, mode of propagation, probability of occurrence and magnitude can not be a certain.
- The Net magnitude of a Random errors in an instrument or a measurement system is generally negligible.
- The Magnitude of Random error can approximated by statistical method such as Mean and standard deviation.
- If the deviation of the measured value from true value is specified by the manufacturer himself, this deviation is known as the Limiting error or the guarantee error.

$$* \quad A_a = A_s \pm \delta A \quad \text{----- (1)}$$

where, A_a = measured value (or Actual value)

A_s = True value (or Nominal or stated value)

δA = Absolute limiting error (also denoted by ϵ_a)

↳ As δA does not signify any useful information, hence it is always specified in relation to the true value as the relative limiting Error (ϵ_r).

$$\therefore \epsilon_r = \frac{\delta A}{A_s} \quad \text{----- (2)}$$

\therefore % relative limiting error will be,

$$\% \epsilon_r = \frac{\delta A}{A_s} \times 100 \quad \text{----- (3)}$$

By taking expression (2), we have:

$$\delta A = A_s \epsilon_r$$

Substituting the above in (1), we have

$$A_a = A_s \pm \epsilon_r A_s$$

$$A_a = A_s (1 \pm \epsilon_r) \quad \text{----- (4)}$$

Expression (1) can now be written as,

$$\delta A = A_a - A_s$$

Substituting the above in (3), we have

$$\% \epsilon_r = \frac{A_a - A_s}{A_s} \times 100$$

$$\% \text{ Relative limiting Error} = \frac{MV - TV}{TV} \times 100 \quad \text{----- (5)}$$

where, MV = Measured value.

TV = True value.

↳ Note :- The absolute value of the error (δA) is always a constant value.

- As the relative error decreases with the increase in deflection, it is always advisable to take the reading of an indicating instrument at the last $\frac{1}{3}$ rd of the scale (farther end of the scale (or) nearer to the full scale deflection (FSD)).

Q) Assertion (A) :- It is always desirable to take the reading of an indicating instrument very close to the FSD.

Reason (R) :- Accuracy of the indicating instrument is max at full scale deflection (FSD) and the error increases as the reading come closer at the beginning of the scale.

Ans \Rightarrow (option-a)

Q) A voltage of 2.70 V is measured by indicating instrument having the scale range of 0 to 5 V. If the instrument reads 2.65 V, then the absolute error in the measurement is :-

(a) + 2.0 V

(c) + 0.05 V

(b) - 2.0 V

(d) - 0.05 V

Ans \Rightarrow Measured value = 2.70 V = A_a

True value = 2.65 V = A_s

$$\delta A = A_a - A_s$$

$$= 2.70 - 2.65 = +0.05 \text{ V (option-c)}$$

Q) A voltmeter has a True value of 1.50 V and an instrument of scale range of (0 to 2) V shows voltage of 1.46 V, the values of the absolute error and correction required are :-

- (a) 0.4V and -0.8V
 (b) -0.04V and +0.04V
 (c) 0.08V and +0.04V
 (d) None of the above.

Ans $A_a = 1.46 \text{ V}$, $A_s = 1.50 \text{ V}$.

$$\begin{aligned} \delta A &= A_a - A_s \\ &= 1.46 - 1.50 \\ &= -0.04 \text{ V} \end{aligned}$$

correction required = $-(\delta A)$
 $= +0.04 \text{ V}$ (option-b).

Q) A (0 to 10) Ammeter has a guarantee accuracy of 1% of FSD. The limiting error while measuring 2.5 A is :-

- (a) $\pm 1\%$ (c) $\pm 4\%$
 (b) $\pm 2\%$ (d) None of the above.

Ans (0 to 10) A $\pm 1\%$ of FSD.

$$\Rightarrow \frac{\delta A}{A_s} \times 100 \text{ when } A_s = 10 \text{ A is } 1.$$

$$\text{So, } \frac{\delta A}{10} \times 100 = 1.$$

$$\Rightarrow \delta A = \frac{1}{10} = \pm 0.1 \text{ A}$$

as δA is a constant for all values from 0 to 10 A.

$$\therefore \frac{\delta A}{A_s} \times 100 \text{ when } A_s = 2.5 \text{ A is } = ?$$

$$\text{So, } \% \text{ Er} = \frac{0.1}{2.5} \times 100 = \pm 4\% \quad (\text{Ans}) \rightarrow (\text{option-c}).$$

Q) A (0 to 300) V voltmeter has a guarantee accuracy of $\pm 2\%$ of FSD. If the voltage measured by the instrument is 180 V, then the limiting error will be :-

- (a) less than 2% (b) 2% (c) 4% (d) 6%

- (b) greater than 2% but less than 3%
- (c) less than 4% but greater than 3%
- (d) 4%

Ans \Rightarrow (0 to 300)V \pm 2% of FSD

i.e. $\frac{\delta A}{A_s} \times 100$ when $A_s = 300$ is 2.

$\therefore \frac{\delta A}{300} \times 100 = 2 \Rightarrow \delta A = \pm 6V$.

as $\delta A = \pm 6V$ is a constant value for voltage to range from 0 to 300.

$\frac{\delta A}{A_s} \times 100$ when $A_s = 180V$ is = ?

$\frac{6}{180} \times 100 = 3.33\%$ (Ans) . (option-c).

Q) A voltmeter has a range of 0 to 20V and manufacturer notes its accuracy as $\pm 1\%$ of FSD. Match List-I (voltage) with List-II (% Er).

<u>List-I</u>	<u>List-II</u>
(A) 2% V	(a) 4%
(B) 5 V	(b) 10%
(C) 10 V	(c) 2%
(D) 20 V	(d) 1%

Ans \Rightarrow (A) \rightarrow (b) ; (B) \rightarrow (a) ; (C) \rightarrow (c) ; (D) \rightarrow (d).

Q) The value of capacitance of a capacitor is specified as $1\mu F \pm 5\%$ by the manufacturer. The limiting b/w which the capacitance values can be guarantee are :-

- (a) 0.95 μF
- (b) 1.05 μF
- (c) 0.95 to 1.05 μF
- (d) 1 μF to 1.05 μF

Ans $\Rightarrow C = 1 \mu F \pm 5\%$

Here, $\frac{\delta A}{A_s} \times 100 = 5$

or, $\epsilon_r = \frac{\delta A}{A_s} = \frac{5}{100} = 0.05$

We know,

$$A_a = A_s (1 \pm \epsilon_r)$$

Lower Range,

$$A_a = 1(1 - 0.05) = 0.95 \mu F$$

upper Range,

$$A_a = A_s(1 + \epsilon_r) = 1(1 + 0.05) = 1.05 \mu F \quad (\text{option-c}).$$

Q) A (0 to 300)V voltmeter has a guarantee accuracy of $\pm 2\%$ of FSD. What could be the Range in which its reading can be guaranteed for a voltage of 30V to be measured.

Ans \Rightarrow Method-I :- (for conventional)

$$(0 \text{ to } 300) V \pm 2\% \text{ of FSD}$$

i.e. $\frac{\delta A}{A_s} \times 100 = 2$ when $A_s = 300V$

$$\therefore \frac{\delta A}{300} \times 100 = 2 \Rightarrow \delta A = \pm 6V$$

As the voltmeter measure 30V

$\therefore \epsilon_r$ when $A_s = 30$ will be,

$$\epsilon_r = \frac{\delta A}{A_s} = \frac{6}{30} = 0.2$$

The range in which the reading can be guaranteed are,

Lower Range, $A_a = A_s(1 - \epsilon_r)$
 $= 30(1 - 0.2) = 24V$

upper Range, $A_a = 30(1 + 0.2) = 36V$

* Method-II :- (for objective)

$$\frac{\delta A}{A_s} \times 100 = 2 \text{ when } A_s = 300$$

$$\therefore \delta A = \pm 6V$$

As δA is constant,

$$\text{Lower Range} = 30 - 6 = 24V$$

$$\text{upper Range} = 30 + 6 = 36V$$

Q) If the measured value of capacitor is $205.3 \mu f$ and its true value is $201.4 \mu f$. The % relative error is :-

(a) 1.94%

(c) 1.89%

(b) 24.64%

(d) $\pm 39\%$

Ans \Rightarrow % Er = $\frac{MV - TV}{TV} \times 100 = \left(\frac{205.3 - 201.4}{201.4} \right) \times 100 = 1.94\%$ (a)

Q) The measured value of the voltage across a resistor is 80V and true value is 79V. What is the % error and relative accuracy in measurement.

(a) 1.265% and 98.735%

(c) 2.625% and 97.375%

(b) 1% and 99%

(d) 1.25% and 98.75%

Ans \Rightarrow % Er = $\left(\frac{80 - 79}{79} \right) \times 100 = 1.265\%$

Relative accuracy = $100 - 1.265 = 98.735\%$ (a).

* Q) A symmetrical square wave is applied to an average reading voltmeter with its scale calibrated in terms of the rms value of a sine wave. The % error in the reading of the instrument is :-

(a) +111%

(c) -11%

(b) +11%

(d) -111%

Ans \Rightarrow If scale is calibrated in terms of the RMS value of a sine wave, then a multiplication factor of 1.11 is used as,

$$\text{Form factor} = \frac{E_{\text{rms}}}{E_v} = 1.11 \quad (\text{for sine wave})$$

$$\text{or, } E_{\text{rms}} = 1.11 E_v$$

Any signal is applied to this instrument, the o/p will be multiplied by 1.11.

But when a symmetrical square wave is applied,

$$\text{Form factor} = \frac{E_{\text{rms}}}{E_v} = 1$$

$$\text{or, } E_{\text{rms}} = E_v$$

$$\therefore MV = 1.11 E_v$$

$$\cdot TV = E_v$$

$$\begin{aligned} \therefore \% \text{ Error} &= \frac{MV - TV}{TV} \times 100 \\ &= \left(\frac{1.11 E_v - E_v}{E_v} \right) \times 100 \\ &= +11 \% \quad (\text{option - b}) \end{aligned}$$

\Rightarrow Limiting error in combination of quantities :-

- Sum or difference of two or more than two parameters,

$$\text{Let, } x = x_1 + x_2 + x_3$$

$$\delta x = \delta x_1 + \delta x_2 + \delta x_3$$

specifying the above expression of its relative value,

$$\left[\frac{\delta x}{x} = \pm \left\{ \frac{x_1}{x} \frac{\delta x_1}{x_1} + \frac{x_2}{x} \frac{\delta x_2}{x_2} + \frac{x_3}{x} \frac{\delta x_3}{x_3} \right\} \right]$$

Q) Three resistor has the following reading $R_1 = 37\Omega \pm 5\%$, $R_2 = 75\Omega \pm 5\%$, $R_3 = 50\Omega \pm 5\%$, the limiting error in Ω or % of these resistances. connected in series is,

Ans \rightarrow (Method-1)
 $R = R_1 + R_2 + R_3$
 $= 75 + 50 + 37$
 $= 162\Omega$

$\therefore \frac{\delta R_1}{R_1} \times 100 = 5$ when $R_1 = 37$

$\frac{\delta R_1}{37} \times 100 = 5 \Rightarrow \delta R_1 = \frac{5 \times 37}{100} = \pm 1.86\Omega$

$\therefore \frac{\delta R_2}{75} \times 100 = 5 \Rightarrow \delta R_2 = \frac{5 \times 75}{100} = 3.75\Omega$

$\therefore \frac{\delta R_3}{50} \times 100 = 5 \Rightarrow \delta R_3 = \frac{5 \times 50}{100} = 2.5\Omega$

Now, $\delta R = \delta R_1 + \delta R_2 + \delta R_3$
 $= 1.86 + 3.75 + 2.5$
 $= 8.1\Omega$

% Er = $\frac{\delta R}{R} \times 100$
 $= \frac{8.1}{162} \times 100 = \pm 5\%$

* Method-2 :-

$\frac{\partial R}{R} = \frac{R_1}{R} \frac{\partial R_1}{R_1} + \frac{R_2}{R} \frac{\partial R_2}{R_2} + \frac{R_3}{R} \frac{\partial R_3}{R_3}$

Given, $R = 162\Omega$

$\frac{\partial R_1}{R_1} = \frac{\partial R_2}{R_2} = \frac{\partial R_3}{R_3} = \frac{5}{100} = 0.05$

$\frac{\partial R}{R} = \pm \left\{ \frac{37}{162} \times 0.05 + \frac{75}{162} \times 0.05 + \frac{50}{162} \times 0.05 \right\}$
 $= \pm 0.05$

$\frac{\partial R}{R} \times 100 = \pm 5\%$ (Ans)

Q) A four dial decade resistance box as,

decade A $\rightarrow x_1 \times 1000 \pm 0.1\%$

decade B $\rightarrow x_2 \times 1000 \pm 0.1\%$

decade C $\rightarrow x_3 \times 10 \pm 0.5\%$

decade D $\rightarrow x_4 \times 1 \pm 1.0\%$

If the terminal resistance of the decade resistance is 4639Ω .
Find the % error and the range of resistance value in which the resultant can be guaranteed.

Ans \rightarrow

⊙ Product or Quotient of two or more than two parameter :-

Let $x = x_1 \cdot x_2 \cdot x_3$.

Applying Log on both side,

$$\log x = \log x_1 + \log x_2 + \log x_3$$

Differentiate, we have;

$$\left[\frac{\delta x}{x} = \pm \left\{ \frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \frac{\delta x_3}{x_3} \right\} \right]$$

Q) The arms of the wien stone bridge is shown, for the balance condition of the bridge, the tolerance value of R_4 is,

Ans \rightarrow (a) $50 \Omega \pm 2\%$

(c) $50 \Omega \pm 5\%$

(b) $50 \Omega \pm 3\%$

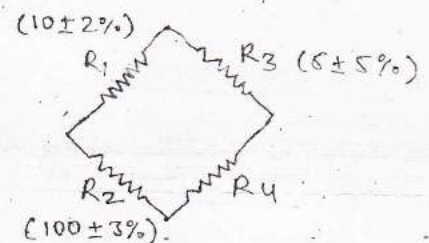
(d) $50 \Omega \pm 10\%$

Ans \rightarrow
$$R_4 = \frac{R_2 R_3}{R_1} = \frac{100 \times 5}{10} = 50 \Omega$$

$$\therefore \frac{\delta R_4}{R_4} = \pm \left\{ \frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3} + \frac{\delta R_1}{R_1} \right\} \times 100$$

$$= \pm \{ 3 + 5 + 2 \}$$

$$\frac{\delta R_4}{R_4} \times 100 = \pm 10\%$$



Q) A resistance is measured by the voltmeter-ammeter method, The voltmeter reading is 80V on 100V scale and ammeter

reading is 40 mA on 50 mA scale. If both the meter are guaranteed to an accuracy of $\pm 0.8\%$ of FSD, The limits with in which the resistance is measured is,

(a) $\pm 32 \Omega$

(b) $\pm 40 \Omega$

(c) $\pm 20 \Omega$

(d) None of the above.

Ans $\Rightarrow (0-100)V \pm 0.8\%$

$(0-50)mA \pm 0.8\%$

Reading of meter,

Voltmeter = 80V

Ammeter = 40 mA

Here, $R = \frac{V}{I} = \frac{80}{40 \times 10^{-3}} = 2000 \Omega$

from the given value,

$\frac{\delta V}{V} \times 100$ when $V = 100$ is 0.8

$\frac{\delta V}{100} \times 100 = 0.8 \Rightarrow \delta V = \pm 0.8 V$

If $V = 80 V$ then,

$\frac{\delta V}{V} \times 100 = \frac{0.8}{80} \times 100 = 1\%$

$\frac{\delta I}{I} \times 100$ when $I = 50 mA$ is 0.8

$\frac{\delta I}{50} \times 100 = 0.8 \Rightarrow \delta I = \pm 0.4 mA$

if $I = 40 mA$

$\frac{\delta I}{I} \times 100 = \frac{0.4}{40} \times 100 = 1\%$

As, $R = V/I$

$\frac{\delta R}{R} = \pm \left\{ \frac{\delta V}{V} \pm \frac{\delta I}{I} \right\} = 1 + 1 = 2\%$; 2% of 2000 is 40 Ω

⊙ Power of a factor :-

Let $x = x_1^n$

Applying Log and differentiate,

$$\frac{\delta x}{x} = \pm n \frac{\delta x_1}{x_1}$$

⊙ Composite function (factors) :-

Let, $x = x_1^n \cdot x_2^m$.

Apply log on both side and differentiate,

$$\left[\frac{\delta x}{x} = \pm \left\{ n \cdot \frac{\delta x_1}{x_1} + m \cdot \frac{\delta x_2}{x_2} \right\} \right]$$

Q) The power in ckt is measured in term of current flowing through a resistance the current is measured with accuracy of 1.5%, the tolerance band of resistor is $\pm 0.5\%$, the % limiting error in the measured of power is :-

(a) $\pm 1.5\%$

(c) $\pm 2\%$

(b) $\pm 3.5\%$

(d) $\pm 2.5\%$

Ans $\Rightarrow P = I^2 R$

$$\frac{\delta P}{P} \times 100 = \pm \left\{ 2 \frac{\delta I}{I} + \frac{\delta R}{R} \right\} \times 100$$

$$\frac{\delta P}{P} \times 100 = \pm \left\{ 2 \times 1.5 + 0.5 \right\} \dots$$

$$= \pm 3.5\% \quad (\text{option-b})$$

Q) A $470 \Omega \pm 10\%$ resistor, as a potential difference of 12 across its terminal. If the voltage is measured with an accuracy of $\pm 6\%$. Determine the power dissipated by the resistor and % error in the power.

Ans $\Rightarrow P = \frac{V^2}{R} = \frac{12 \times 12}{470} = 0.306$

$$\frac{\delta P}{P} \times 100 = \pm \left\{ 2 \frac{\delta V}{V} + \frac{\delta R}{R} \right\} \times 100$$

$$= 2 \times 6 + 10 = 22\%$$

$$P = 0.306 \pm 22\%$$

Conv.

Q) A quantity x is calculated using the formula,

$$x = \frac{P-Q}{R}$$

and measured values are, $P=9$, $Q=6$ and $R=0.5$.

Assume that measured values are independent and absolute maximum error in the measurement of each of these quantities is ϵ . Calculate the value of absolute maximum error in the measurement of X .

Ans → From the given data,

$$X = \frac{P-Q}{R} = \frac{9-6}{0.5} = 6$$

let $Y = P - Q = 9 - 6 = 3$.

From the expression for sum and difference,

$$\begin{aligned} \frac{\delta Y}{Y} &= \pm \left\{ \frac{P}{Y} \cdot \frac{\delta P}{P} + \frac{Q}{Y} \cdot \frac{\delta Q}{Q} \right\} \\ &= \pm \left\{ \frac{9}{3} \cdot \frac{\epsilon}{9} + \frac{6}{3} \cdot \frac{\epsilon}{6} \right\} = \pm 2\epsilon/3 \end{aligned}$$

Now, we have, $X = \frac{Y}{R}$.

From the expression for product or quotient,

$$\frac{\delta X}{X} = \pm \left\{ \frac{\delta Y}{Y} + \frac{\delta R}{R} \right\}$$

$$\frac{\delta X}{6} = \pm \left\{ \frac{2\epsilon}{3} + \frac{\epsilon}{0.5} \right\}$$

$$\delta X = \left(\frac{2\epsilon}{3} + 2\epsilon \right) \times 6 = \pm 16\epsilon$$

⊙ Known Error :-

- If the effect of the error is known, then this error is defined as a known error.

Q) Current was measured during a test as 30.4 A flowing in a resistor of 0.105Ω . It was discovered later that the Ammeter was reading low by 1.2% and the marked resistance is high by 0.3%. Find the true power as a % of the power originally calculated.

Ans → True power as a % of power originally calculate,

$$= \frac{\text{True Power}}{\text{Calculated power}} \times 100 \quad \dots (1)$$

Given that, $I = 30.4 \text{ A}$

$$R = 0.105 \Omega$$

$$\therefore P = I^2 R$$

$$= (30.4)^2 \times (0.105)$$

$$= 97.036 \text{ Watt.}$$

(a) As the ammeter reads lower by 1.2%, the actual value is 1.2% increases.

$$\text{given, } \frac{\delta I}{I} \times 100 = 1.2$$

$$\text{or, } \epsilon_I = \frac{\delta I}{I} = \frac{1.2}{100} = 0.012$$

Actual value of current, $= A_s (1 + \epsilon_I)$

$$= 30.4 (1 + 0.012)$$

$$= 30.765$$

(b) As the marked resistance was 0.3% increases, the actual value is 0.3% decreases,

$$\text{given, } \frac{\delta R}{R} \times 100 = 0.3$$

$$\epsilon_R = \frac{\delta R}{R} = \frac{0.3}{100} = 0.003$$

Actual value of R, $A_a = A_s (1 - \epsilon_R)$

$$= 0.105 (1 - 0.003) = 0.1046$$

The actual value of power will be,

$$P = I^2 R$$

$$= (30.765)^2 \times 0.1046$$

$$= 99.0023$$

$$\text{We have, } \frac{99.0023}{97.036} \times 100 = 102.02\%$$

◎ UNCERTAINTY ANALYSIS :-

- If the deviation of the measured value from true value is specified for a single or uni-sample of a data either in terms of degree of confidence of user or the odds against which the measurement was taken, then this deviation

defined as uncertainty involved in measurement of that parameter.

Let, $x = f \{x_1, x_2, x_3, \dots, x_n\}$.

and if $w_{x_1}, w_{x_2}, \dots, w_{x_n}$ are uncertainty in x_1, x_2, \dots, x_n , then the absolute uncertainty in measurement of x will be,

$$w_x = \pm \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 w_{x_1}^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 w_{x_2}^2 + \dots + \left(\frac{\partial x}{\partial x_n}\right)^2 w_{x_n}^2}$$

Q) Two resistance are connected in series the value of Resistance are $R_1 = 100 \pm 0.1 \Omega$ and $R_2 = 50 \pm 0.03 \Omega$. Calculate the uncertainty in the combined Resistance?

Ans $R = R_1 + R_2 = 100 + 50 = 150$

$$w_R = \pm \sqrt{\left(\frac{\partial R}{\partial R_1}\right)^2 w_{R_1}^2 + \left(\frac{\partial R}{\partial R_2}\right)^2 w_{R_2}^2}$$

$$\frac{\partial R}{\partial R_1} = \frac{\partial (R_1 + R_2)}{\partial R_1} = 1$$

$$\frac{\partial R}{\partial R_2} = \frac{\partial (R_1 + R_2)}{\partial R_2} = 1$$

$$w_R = \pm \sqrt{1^2 \cdot (0.1)^2 + 1^2 \cdot (0.03)^2} = \pm 0.1044 \Omega$$

Q) Power in a DC ckt is calculated as a product of current drawn and impressed voltage. If the values of current and voltage in measurement are 5.3 A and 110.2 V and uncertainty involved in the measurement being 0.06 A and 0.1 V . Calculate the power dissipated by the load and the uncertainty involved in its measurement.

Ans $I = 5.3 \text{ A}$, $V = 110.2 \text{ V}$

$$P = VI = 110.2 \times 5.3$$

$$\omega_I = 0.06 \text{ A}$$

$$\omega_V = 0.1 \text{ V}$$

$$\begin{aligned} \omega &= \pm \sqrt{\left(\frac{\partial P}{\partial V}\right)^2 \omega_V^2 + \left(\frac{\partial P}{\partial I}\right)^2 \omega_I^2} \\ &= \pm \sqrt{I^2 \times (0.1)^2 + V^2 \times (0.06)^2} \\ &= \pm \sqrt{(5.3)^2 (0.1)^2 + (110.2)^2 (0.06)^2} \\ &= \pm 6.62 \text{ Watts} \end{aligned}$$

Q) A resistor has a nominal resistance $10 \Omega \pm 0.1\%$ a voltage is applied across the resistor and the power consumed is calculated as $P = E^2/R$. Calculate the % uncertainty when the measured value $E = 100 \pm 1\%$.

Ans

$$R = 10 \pm 0.1\%$$

$$E = 100 \text{ V} \pm 1\%$$

$$P = E^2/R$$

$$\begin{aligned} \omega_P &= \pm \sqrt{\left(\frac{\partial P}{\partial E}\right)^2 \omega_E^2 + \left(\frac{\partial P}{\partial R}\right)^2 \omega_R^2} \\ &= \sqrt{\left(\frac{2E}{R}\right)^2 \omega_E^2 + \left(-\frac{E^2}{R^2}\right)^2 \omega_R^2} \end{aligned}$$

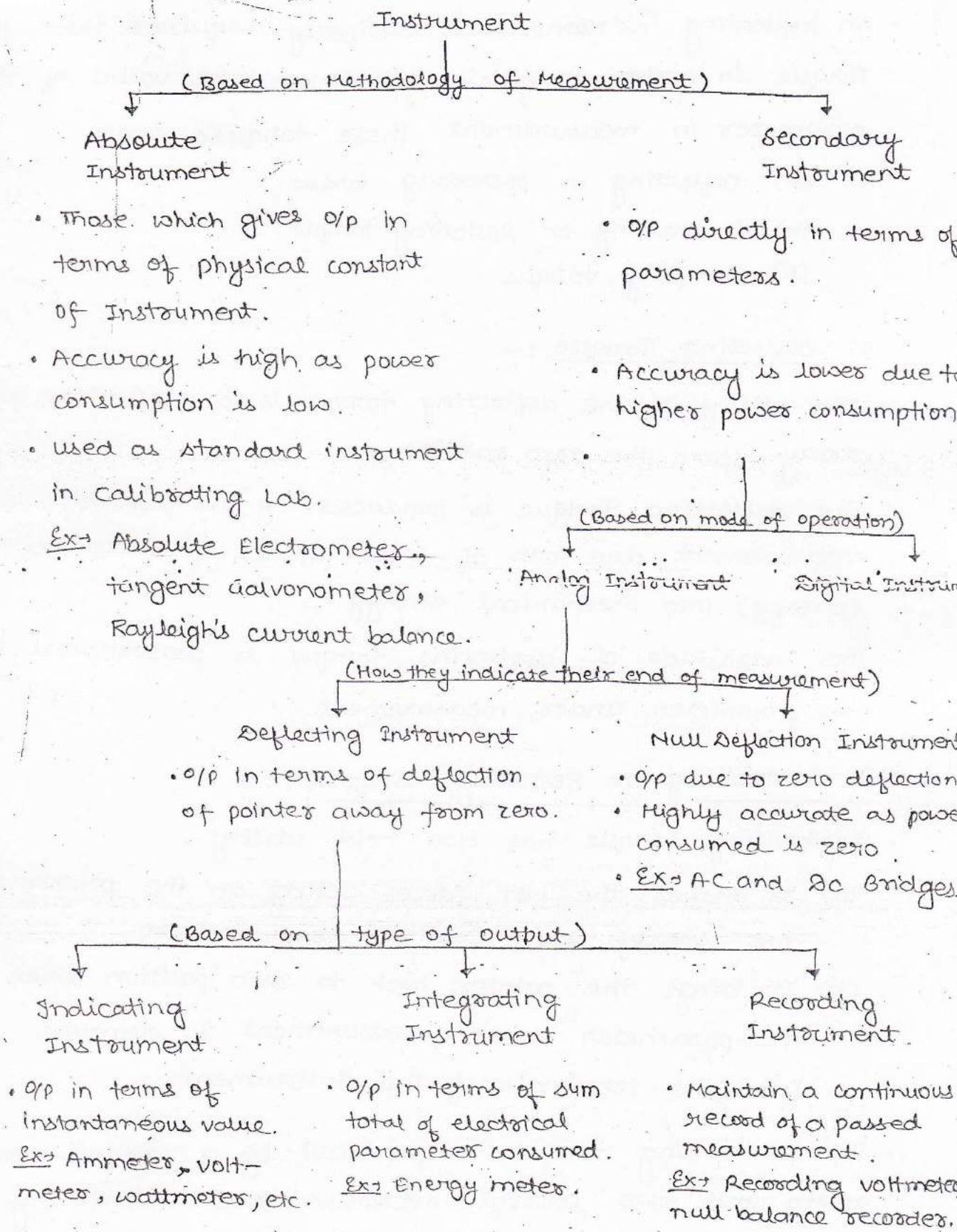
Relative uncertainty can be expressed as,

$$\begin{aligned} \frac{\omega_P}{P} &= \pm \sqrt{\left(\frac{4E^2}{R^2} \omega_E^2 + \frac{E^4}{R^4} \omega_R^2\right)} \times \frac{R}{E^2} \\ &= \pm \sqrt{4 \left(\frac{\omega_E}{E}\right)^2 + \left(\frac{\omega_R}{R}\right)^2} \end{aligned}$$

$$\begin{aligned} \text{Given, } \frac{\omega_E}{E} \times 100 &= 1 & \left| \frac{\omega_R}{R} \times 100 &= 0.1 \\ \frac{\omega_E}{E} &= 0.01 & \frac{\omega_R}{R} &= 0.001 \end{aligned}$$

$$\frac{\omega_P}{P} = \pm \sqrt{4(0.01)^2 + (0.001)^2} = \pm 0.02002$$

① Classification of Instrument :-



⊙ Essential of Indicating Instrument :-

- An indicating instrument is basically required three forces/Torque in order to efficiently indicate the value of the parameter in measurement these torques are:-

- (a) Deflecting or operating torque.
- (b) Controlling or Restoring torque.
- (c) Damping torque.

(a) Deflecting Torque :-

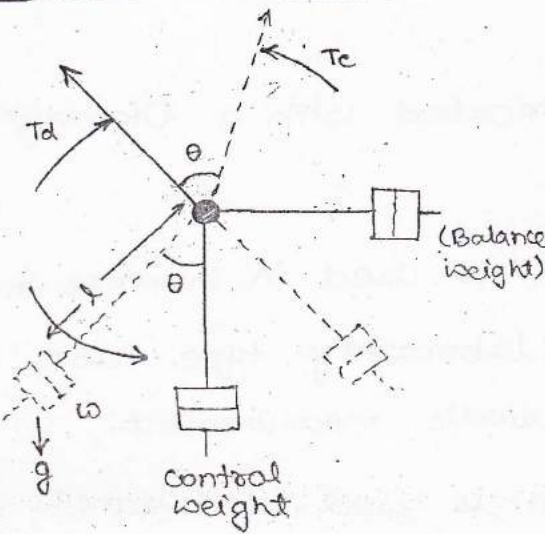
- The utility of the deflecting torque is to deflect the pointer away from the zero position.
- The deflecting torque is produced by the parameter under measurement due one of those effect of electric current (energy) into mechanical energy.
- The magnitude of deflecting torque is proportional to the parameter under measurement.

(b) Controlling or Restoring torque :-

- Controlling torque has two hold utility,
 - (a) It makes the net forces acting on the pointer at the steady state position equal to zero.
 - (b) It brings the pointer back to zero position when the parameter under measurement is removed from the terminals of the Instrument.
- The controlling torque is produced by a control mechanism and two control mechanism are used in any indicating instrument are :-

- (i) Spring control mechanism.
- (ii) Gravity control mechanism.

* Gravity Control



$$T_c = w l \sin \theta \quad (\text{N/m})$$

w = control weight (kg)
 l = distance (meter)
 theta = deflection, (radian)

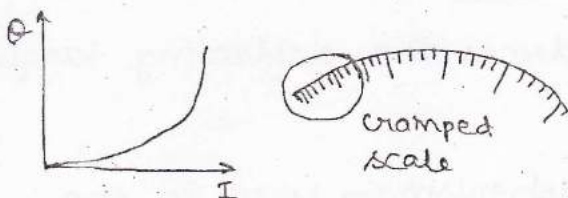
$$T_c = K \sin \theta$$

$$T_c \propto \sin \theta$$

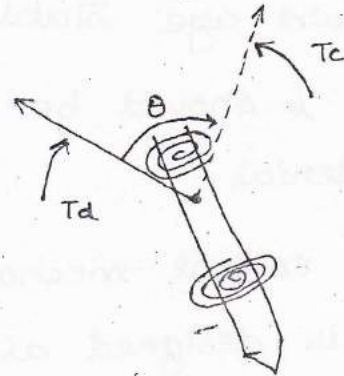
Assume $T_d \ll I$ and in this case $T_c \propto \sin \theta$.

∴ at steady state position

$$T_c = T_d \text{ or } I \propto \sin \theta$$



Spring Control



$$T_c = \frac{E b t^3}{12 l} \theta \quad (\text{Nm})$$

E = Young's Modulus (N/m²)
 b = width (cm)
 t = thickness (cm)
 l = length (cm)
 theta = Angular deflection (cm)

As E, b, t and l are constant

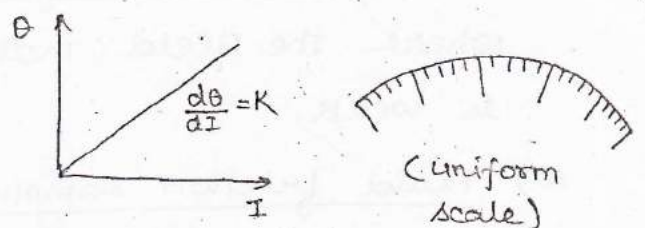
$$T_c = K \theta \quad (\text{Nm})$$

$$T_c \propto \theta$$

Assume $T_d \ll I$ and in this case $T_c \propto \theta$ at steady state position.

$$T_c = T_d$$

$$\& \theta \propto I$$



- The essential characteristics of a spring which is used as a control mechanism are :-
 - (a) It should undergoes minimal mechanical fatigue (it should age slowly).
 - (b) It should be fabricated with a Diamagnetic material.
- Spring control mechanism is used in instance the instrument is designed as a laboratory type table top instrument intended for periodic measurement.
- Gravity control mechanism is used in instances where the instrument is designed as a vertical placed panel mounted instrument intended for continuous monitoring.

(3) The Damping Torque :-

- The utility of the damping Torque is to damped the oscillation of the pointer at steady state position.
- The damping torque is produced by a damping mechanism and various damping mechanism used are :-
 - (a) Eddy current damping mechanism :- used in a instance where the field the produces the deflecting torque is strong.
 - (b) Air friction damping mechanism → used in instances where the field that produces the deflecting torque is weak.
 - (c) Fluid friction damping mechanism :- used in the

Instrument has low sensitivity.

- The damping mechanism of an indicating instrument travel represent its time response and all the indicating instrument are slightly underdamped by nature.
- The typical value of damping constant 'G' in an indicating instrument is chosen in b/w 0.6 to 0.8.

Q) Assertion (A) :- The middle of an indicating instrument attain a position where deflecting and controlling torque acting on a moving system are equal and opposite.

Reason (R) :- The oscillation of the middle are damped by the damping mechanism.

Ans \rightarrow (option-b)

Q) Assertion (A) :- The PMMC type of indicating instrument are always damped.

Reason (R) :- Critical damped system comes to steady state without oscillation.

Ans \rightarrow A \rightarrow ~~F~~ False, R \rightarrow True (option-d)

Q) Assertion (A) :- A Gravity control instrument has a crowded scale.

Reason (R) :- In a Gravity control instrument, the controlling torque is inversely proportional to the deflecting angle θ .

Ans \rightarrow (A) \rightarrow True, (R) \rightarrow False (because $\propto \sin\theta$) (option-c)

Q) A basic PMMC instrument produces a torque of 16×10^{-6} Nm

for a deflection of 120° . If the length and ~~the~~ width of the spring are doubled and its thickness is half, the torque produced by the spring for the same deflection is:-

(a) 8×10^{-6} Nm

(c) 4×10^{-6} Nm

(b) 16×10^{-6} Nm

(d) 2×10^{-6} Nm

Ans) $T_d = 16 \times 10^{-6}$ Nm

For $\theta = 120^\circ$

at steady state position, $T_c = T_d$.

$\therefore T_c = 16 \times 10^{-6}$ Nm

$$\frac{Ebt^3}{12l} \cdot \theta \rightarrow 16 \times 10^{-6} \text{ Nm}$$

$$\frac{E(2b)(t/2)^3}{12(2l)} \cdot \theta \rightarrow \frac{1}{8} \times \frac{Ebt^3}{12l} \cdot \theta$$

$$= \frac{1}{8} \times 16 \times 10^{-6}$$

$$= 2 \times 10^{-6} \text{ Nm (option-d)}$$

Q) A spring control instrument uses a phosphor bronze spring to produce a controlling torque. If the ratio of length of the spring and its thickness is 3000 for a deflection of 90° . What should be this ratio if the scale is extended to 120° .

Ans (a) 4000

(c) 2000

(b) 2250

(d) None of the above.

Ans \Rightarrow In change in a angular deflection can be made possible by a proportionate change in the l/t ratio, ~~thus~~ as

$$T_c \propto \theta$$

$$\therefore \frac{1}{2} \rightarrow 3000$$

$$\frac{2}{3} \rightarrow ?$$

$$= \frac{2\pi}{3} \times 3000 \times \frac{2}{\pi} = 4000 \text{ (Option - a)}$$

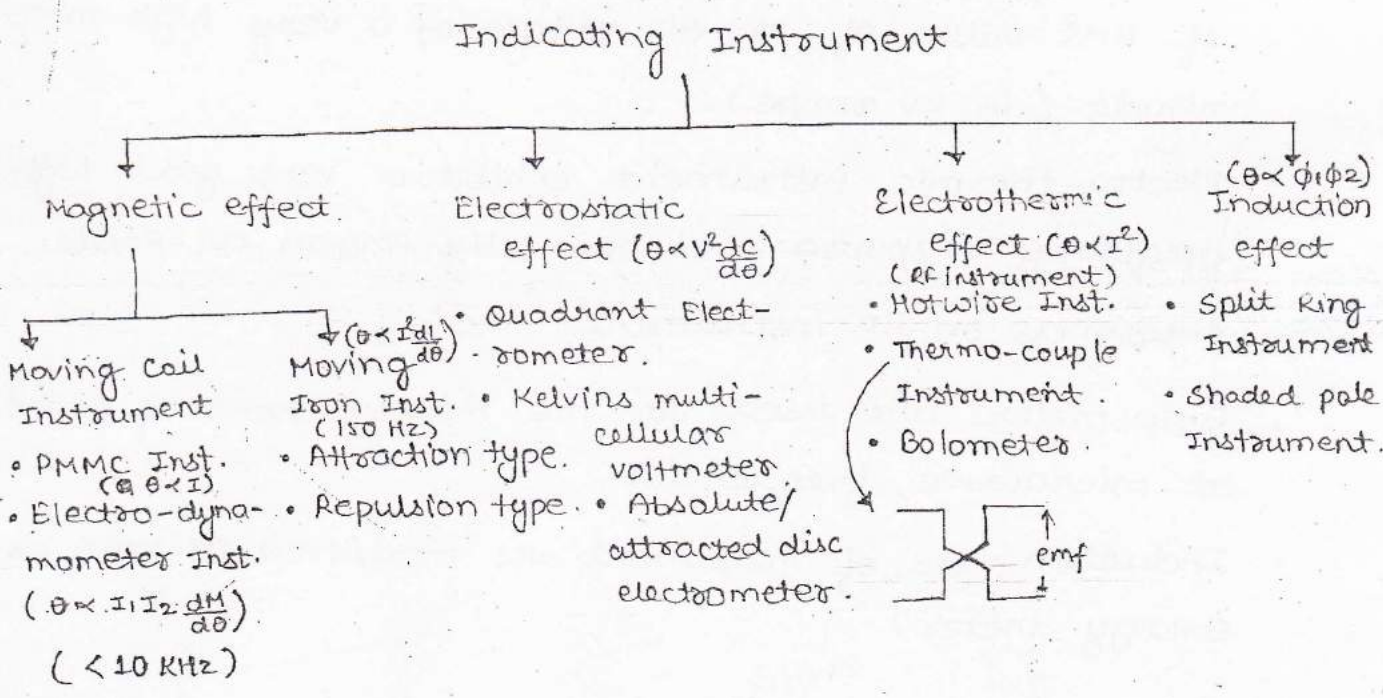
Q) A zero to 5 A PMMC ammeter is supplied with a current of 2.75 A. If the instrument is designed without a control mechanism and its moving system is free to rotate, the pointer of the instrument will :-

- (a) Indicate a value greater than 2.75 but less than 5 A.
- (b) Indicate a value less than 2.75 A.
- (c) Indicate 5 A.
- (d) None of the above.

Ans -> (Option-d)

⊙ Classifying of Indicating Torque Instrument :-

• Indicating instrument are further classified on the basis of the effect of the electric current they utilize to produce the deflecting torque.



- As $\theta \propto I$, a PMMC instrument has a uniform scale.
- If the angular deflection of the instrument is proportional to either the square or product of parameter under measurement, the instrument is said to exhibit a square law response.
- Instrument which exhibit a square law response are characterized by :-
 - (a) A non-uniform scale.
 - (b) Their angular deflection being directly in terms of RMS value of the AC parameter under measurement, where $I_{rms} = \sqrt{I_{dc}^2 + \left(\frac{I_m}{\sqrt{2}}\right)^2}$
- Electro-dynamometer type of instrument are generally used as Watt-meters.
- Electro-static instrument also known as electrometer are used as voltmeter that to for the measurement of RMS value of an AC voltage of a very high magnitude (in KV range).
- Electro-thermic instrument exhibit a very good high frequency response and generally known as Radio-frequency or RF instrument.
- Bolo-meters are used for the measurement of power at microwave frequencies.
- Induction type of instrument are modified to work as Energy meter.

Q)

LIST-I

- (A) PMMC
- (B) Moving Iron
- (C) Thermo-coupled
- (D) Electrostatic type

LIST-II

- (1) Square law type scale.
- (2) very good high freq. Response
- (3) Linear scale over entire range
- (4) Voltmeter.

Ans → (A) → (3) ; (B) → (1) ; (C) → (2) ; (D) → (4)

Q)

LIST-I

- (A) A (0 to 10) mV from a source of internal resistance of 1M Ω .
- (B) Thermo emf of 5mV from a thermo-coupled.
- (C) Supply voltage of 230V and 50 Hz.
- (D) RMS value of a voltage containing DC at ripple of 50 Hz and its harmonics.

LIST-II

- (1) Thermal.
- (2) Moving Iron
- (3) PMMC
- (4) Electronic

Ans → (A) → (4) ; (B) → (3) ; (C) → (2) ; (D) → (1).

* Note :-

The minimum loading effect introduced by an instrument when introduced in a ckt is by Cathode Ray oscilloscope followed by a digital instrument, an electronic instrument and lastly an electro-mechanical instrument (indicating instrument).

Q) A current flowing in 20 Ω resistor is given by $2 + 4\sin 314t$. This current is measured by a hotwire ammeter what is the measured value.

- (a) 2A
- (b) 3.46 A
- (c) 4A
- (d) 2.83 A.

Ans → $i = 2 + 4\sin 314t$

$$I_{rms} = \sqrt{I_D^2 + \left(\frac{I_m}{\sqrt{2}}\right)^2} = \sqrt{2^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \sqrt{12} = 3.46 \text{ (A)}$$

(option-b)

Q) The following voltage is applied to an electrodynamic volt-meter, $V_s = 200 \sin(\omega t) + 40 \sin(3\omega t + 30^\circ) + 30 \cos(5\omega t + 22.5^\circ)$. What is reading of voltmeter.

- (a) 103 V (b) 0 V (c) 145.8 V (d) 425 V.

(Ans) $V_{RMS} = \sqrt{\left(\frac{200}{\sqrt{2}}\right)^2 + \left(\frac{40}{\sqrt{2}}\right)^2 + \left(\frac{30}{\sqrt{2}}\right)^2} = 145.8 \text{ V (option-c)}$

Q) A Rectangular and AC sinusoidal current, each having a peak value of 100 mA are passed through a moving iron ammeter. The meter reading are respectively,

- (a) 0 mA, 0 mA (c) 70.7 mA, 70.7 mA
 (b) 0 mA, 70.7 mA (d) 100 mA, 70.7 mA.

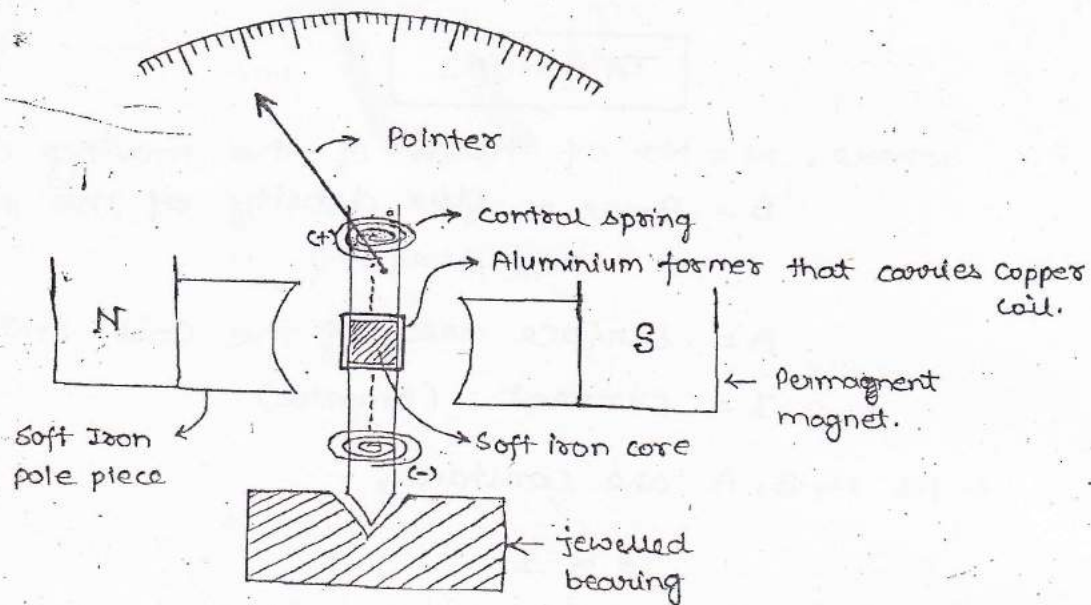
Ans \Rightarrow (option-d)

⊙ Permanent Magnet Moving Coil type Instrument :-
 (D'Arsonval's galvanometer) (PMMC)

- \hookrightarrow construction and working.
- \hookrightarrow Advantage and Disadvantage.
- \hookrightarrow Source of errors.
- \hookrightarrow Application:
 - * voltmeter.
 - * Ammeter.

\Rightarrow Construction and Working :-

- The PMMC instrument is basis its operation on magnetic effect of the electric current.
- The deflecting torque in this instrument is produced on the basis of fact that, "whenever a current carrying conductor placed in a varying magnetic field, the conductor experiences a force which tends to push it away from the direction of magnetic field."



- The fixed system of the instrument consist of a permanent magnet with soft iron pole piece drilled on its pole.
- The use utility of soft iron pole piece is make the field due to permanent magnet, radial in nature.
- The moving system of an instrument consists of spindle on to which, a set of control spring, pointer, a soft iron core and an Aluminium former that carry a copper coil (moving coil) are mounted as shown in fig.
- The soft iron core and the Aluminium former are mounted concentrically wrot a imaginary circle drawn by taking the sectorial surfaces of soft iron pole pieces. ✗
- The controlling torque in this instrument is produced by a spring control mechanism and due to presence of a strong operating field, an Eddy current damping mechanism is used to produced the damping torque.
- The expression that govern the deflecting torque produce in this instrument is given by :-

$$T_d = NBAI \quad \text{Nm}$$

where, N = No. of turns of the moving coil.
 $B = B_{max}$ = flux density of the permanent magnet
 (wb/m^2)

A = surface area of the coil (m^2)

I = current (Ampere)

As N, B, A are constant,

$$T_d \propto I$$

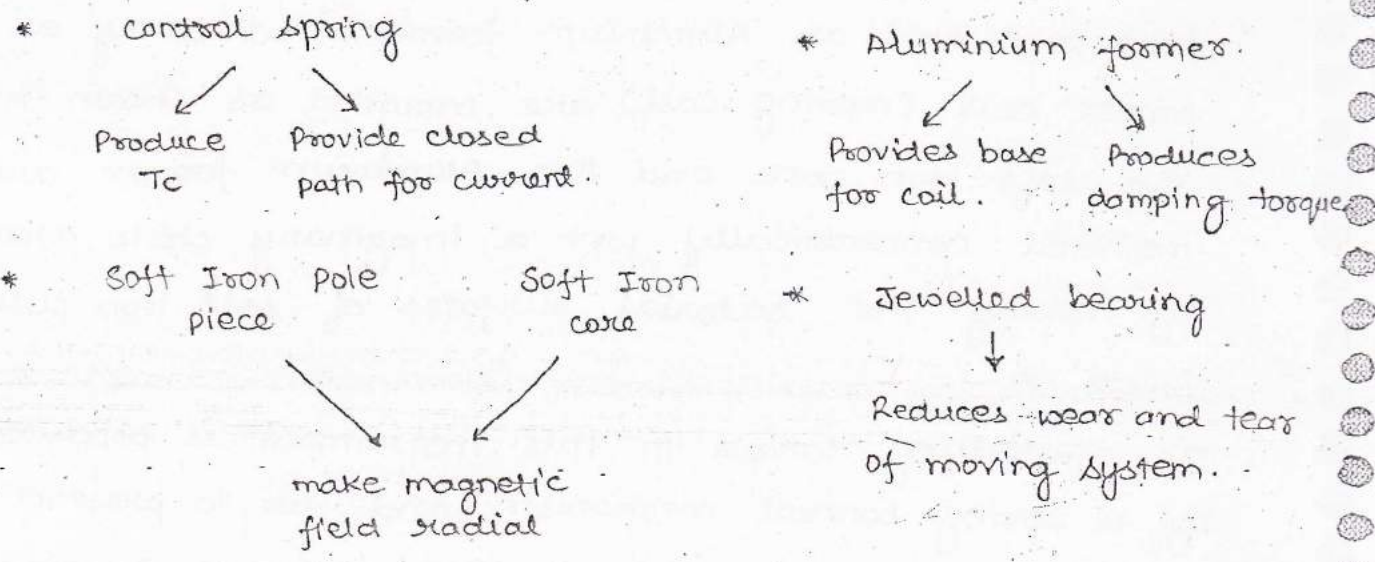
and as a spring control is used,

$$T_c \propto \theta$$

∴ at steady state position, $T_c = T_d$

$$\theta \propto I$$

⇒ Utility of different component :-



Advantage :-

- These instrument have lower power consumption ($25-200 \mu\text{W}$)
- Due to a high Torque to weight ration, these instrument will have a high sensitivity and a high accuracy ($\pm 2\%$ of FSD).

- Due to presence of strong magnetic field, these instrument are not affected by stray magnetic field.
- Since, $\theta \propto I$, instrument gives a uniform scale.

⇒ Disadvantages :-

- As the direction of magnetic field of the permanent magnet does not change with the change in the polarity of the AC parameter under measurement. These instrument are unsuitable for measuring AC current and voltages.

* Notes :-

- (i) If a non-sinusoidal AC is applied, instrument responds to its average value.
- (ii) If a sinusoidal signal is applied for high freq., the pointer vibrates at zero and at low frequency the pointer oscillates.

- As thin and light wires are used to wind, the moving system of the Instrument, these instrument have has a limited current carrying capacity (max. of 100 mA)

⇒ Sources of error :-

- (1) Error due to Ageing of spring, can be compensated by using a pre-aged spring, where pre-aging is done by subjecting the spring to mechanical stress.
- (2) Error due to the Ageing of the permanent Magnet, can be compensated by a using a pre-aged permanent Magnet. Where pre-aging is done by subjecting the permanent magnet to thermal and vibration stress.

(3) Error due to change in the resistance of the spring, due to the heating effect of the electric current, can be compensated by using a spring made up of a suitable material.

* Note :- Essential characteristic of spring, used in as a control mechanism in a PMMC instrument :-

- (i) low resistance.
- (ii) Negligible temperature coefficient of resistance.

⇒ Material :-

- (i) Control Spring → Phosphor bronze.
- (ii) Permanent Magnet → AlNiCo or Alcomax.

Q) ∴ LIST-1

- (A) Former
- (B) Coil
- (C) Core
- (D) Spring

LIST-2

- (1) Produces T_d .
- (2) Provides ~~space~~ for coil.
- (3) Make Magnetic field Radial.
- (4) Provides controlling torque.

Ans ⇒ (A) → (2) ; (B) → (1) ; (C) → (3) ; (D) → (4).

Q) In context of a PMMC instrument, identify the correct matches :-

List-1

- (A) Pair of Spring
- (B) Aluminium former.

List-2

- (1) To provide T_c to θ .
- (2) To provide damping torque.
- (3) Act as base for coil.
- (4) provides current into and out of coil.

Ans ⇒ (A) → 1, 4 .
(B) → 2, 3 .

Q) A PMMC voltmeter is connected across a series combination of a DC voltage source of $V_1 = 2V$ and a AC voltage source of $V_2(t) = 3\sin 314t V$, the meter reads :-

(a) 2V , (b) 5V , (c) $2 + 3\sqrt{2} V$, (d) $\sqrt{17/2} V$

Ans → Since, a high frequency AC is sinusoidal, so its average is equal to zero, hence meter only reads DC i.e. 2V (option-

Q) A high frequency AC signal is applied to a PMMC instrument, if the rms value of AC signal is 2V, then the reading of the instrument will be :-

(a) 2V , (b) $2\sqrt{2} V$, (c) $4\sqrt{2} V$, (d) 0V.

Ans → (option-d) b/c at high frequency, pointer is at 0V.

Q) The PMMC ammeter A shown in the adjoining figure has a range of 0 to 3 mA, when the switch S_1 is opened, the pointer swings to 1mA marks, returns and settle down at 0.9 mA marks. The meter is :-

(a) critically damped, has an coil resistance of 100Ω

(b) " " " " " " of 200Ω

(c) under " " " " " " 100Ω

(d) under " " " " " " 200Ω

Ans → As pointer swing to the 1mA mark, returns and settle down at the 0.9 mA mark :-

(i) The system is under damped.

(ii) The actual current is 0.9 mA

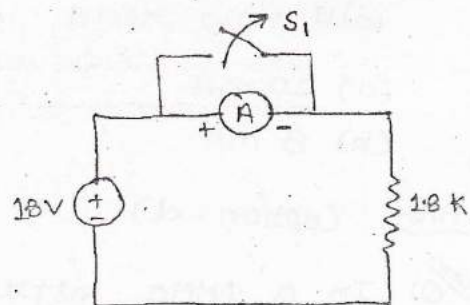
$$R = V/I$$

$$R + R_a = \frac{1.8}{0.9 \times 10^{-3}} = 2000$$

$$1800 + R_a = 2000$$

$$R_a = 200 \Omega$$

(option-d)

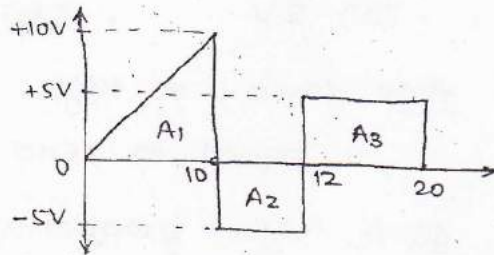


Q) A periodic voltage waveform observe on an oscilloscope across a load is shown. A PMMC meter is connected across the same load reads,

$$\text{Ans} \rightarrow V_{\text{rav}} = \frac{A_1 - A_2 + A_3}{20}$$

$$= \frac{\frac{1}{2} \times 10 \times 10 - 2 \times 5 + 8 \times 5}{20}$$

$$V_{\text{rav}} = 4V \quad (\text{Ans}).$$



Q) An idle diode has been connected across a 10Ω , 100mA centre zero PMMC meter as shown, the meter will read :-

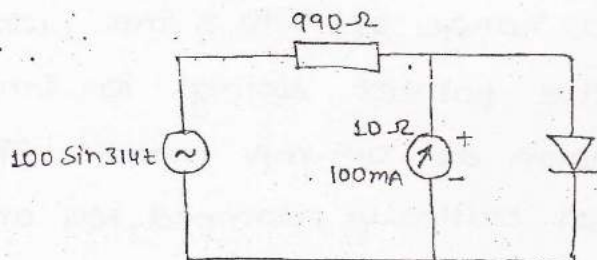
(a) $+100\text{mA}$

(b) $+31.8\text{mA}$

(c) -31.8mA

(d) -67.3mA

Ans \rightarrow H.W.



Q) A $(0\text{ to }10)\text{mA}$ PMMC ammeter reads 4mA in a ckt, its bottom control spring suddenly snaps (break), the meter will now read nearly,

(a) 10mA

(b) 8mA

(c) 2mA

(d) 0

Ans \rightarrow (option - d)

Q) In a PMMC instrument the control spring constant and the strength of the magnetic field decrease by 0.04% and 0.02% resp., due to rise in temperature by 1°C , calculate the % change in the instrument reading for a rise in temp. by 10°C .

Ans → In an PMMC instrument, we know

$$T_d = NBAI \quad Nm$$

As spring control is used,

$$T_c = K\theta$$

at steady state position,

$$T_c = T_d$$

$$\therefore K\theta = NBAI$$

$$\theta = \frac{NBAI}{K} \quad \dots (1)$$

In exp. (1) due to change in 'B' and 'k' if θ increases then reading increases else decreases,

Applying log on both side

$$\log \theta = \log N + \log B + \log A + \log I - \log k.$$

⇒ Application of PMMC instrument :-

* Ammeter :-

↳ Design

↳ Temp. compensation.

↳ Ammeter with multiple Ranges

* voltmeter :-

↳ Design

↳ voltmeter with multiple Range.

↳ voltmeter sensitivity.

↳ voltmeter loading.

* PMMC Ammeter :-

⇒ Ammeter design :-

• As a thin and light wire is used to wind the moving coil of the instrument, a PMMC instrument when used as a

ammeter will have limited range of 100 mA.

- In order to extend this range to measure current beyond 100 mA, a low resistance is connected across the meter movement.
- This low resistance known as the shunt resistance, bypass a measure portion of current through it, thereby protecting the meter movement from damage.

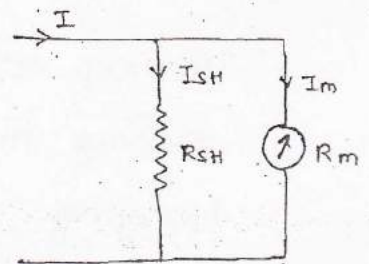
→ In the ckt,

$$I_{SH} R_{SH} = I_m \cdot R_m$$

$$R_{SH} = \frac{I_m R_m}{I_{SH}}$$

here, $I_{SH} = I - I_m$

$$\left\{ R_{SH} = \frac{I_m R_m}{I - I_m} \right\} \dots \dots (1)$$



taking the reciprocal of (1) and multiply by ' R_m ' on both side,

$$\frac{R_m}{R_{SH}} = \frac{(I - I_m) R_m}{I_m R_m}$$

$$\frac{R_m}{R_{SH}} = \frac{I}{I_m} - 1$$

Here, ' m ' is the multiple factor of the shunt, hence

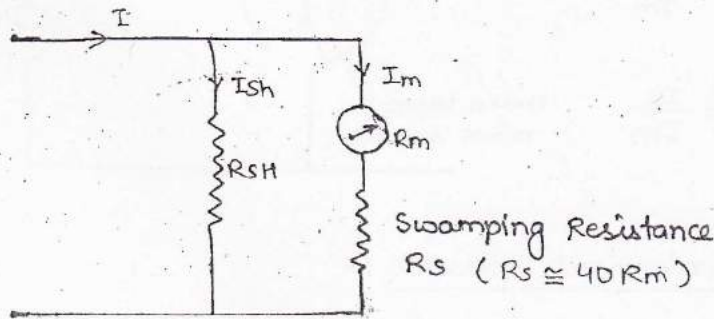
$$\left\{ m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{SH}} \right\} \dots \dots (2)$$

expressed R_{SH} in term of m , we have.

$$\left\{ R_{SH} = \frac{R_m}{(m-1)} \right\} \dots \dots (3)$$

⇒ Temperature Compensation :-

* Note :- The most commonly used material for the fabrication of shunt resistance are Constantan, manganin, Eureka.



- Error due to temp. variation in PMMC ammeter occur due to divergence rate of change of resistances of the shunt ammeter arm.

- In order to compensate, due to temperature variation, a swamping Resistance ($R_s \approx 40R_m$) is connected with in series with meter movement as shown in the figure above. As copper form a small component of series combination shown above, the rate at which the meter arm changes its resistance is mostly due to R_s and negligible due to R_m .

- Thus if R_s and R_{sh} are fabricated with the same material, the rate at which the shunt and meter arm change their resistance would be the same therefore compensating for error, due to temperature variation.

* Note :-

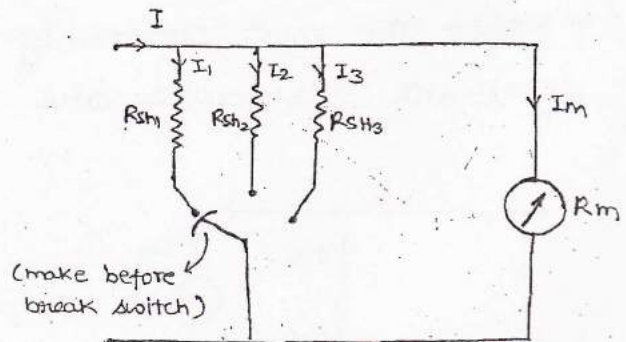
- The most commonly used material for fabrication of swamping resistance in a PMMC ammeter is manganin.

⇒ Ammeter with Multiple Range :-

$$R_{SH1} = \frac{R_m}{(m_1 - 1)} \quad ; \quad m_1 = \frac{I_1}{I_m}$$

$$R_{SH2} = \frac{R_m}{(m_2 - 1)} \quad ; \quad m_2 = \frac{I_2}{I_m}$$

$$R_{SH3} = \frac{R_m}{(m_3 - 1)} \quad ; \quad m_3 = \frac{I_3}{I_m}$$



⇒ Adjustable Shunt or universal Shunt :-

$$R_1 = \frac{R_m}{(m_1 - 1)} \quad ; \quad m_1 = \frac{I_1}{I_m}$$

$$R_2 (I_2 - I_m) = I_m (R_m + R_1 - R_2)$$

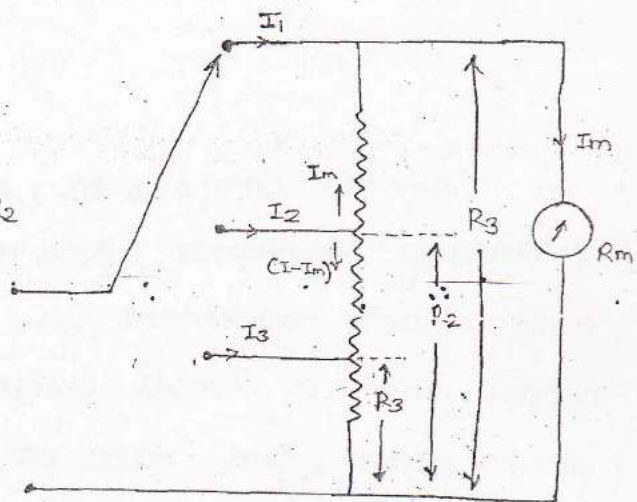
$$I_2 R_2 - I_m R_2 = I_m R_m + I_m R_1 - I_m R_2$$

$$I_2 R_2 = I_m R_m + I_m R_1$$

$$R_2 = \frac{I_m (R_m + R_1)}{I_2}$$

$$R_2 = \frac{R_m + R_1}{m_2} \quad ; \quad m_2 = \frac{I_2}{I_m}$$

$$R_3 = \frac{R_m + R_1}{m_3} \quad ; \quad m_3 = \frac{I_3}{I_m}$$



Q) A 100 μ A ammeter as an internal resistance of 100 Ω . For extending its range for 500 μ A, the shunt resistance required in Ω is :-

- (a) 20 Ω , (b) 22.22 Ω , (c) 25 Ω , (d) 50 Ω

Ans) $I = 500 \mu A$, $I_m = 100 \mu A$

$$m = \frac{I}{I_m} = \frac{500}{100} = 5$$

$$R_m = 100 \Omega$$

$$R_{sh} = \frac{R_m}{(m-1)} = \frac{100}{(5-1)} = 25 \Omega \quad \text{(option-c)}$$

Q) A ~~10A~~ 2A Full scale PMMC type DC ammeter has a voltage drop of 10 mV at 2A. The meter can be converted into 10A Full scale DC ammeter by connecting a:-

- (a) 1.25 mΩ resistor in parallel with meter.
- (b) 1.25 mΩ resistor in series with meter.
- (c) 50 mΩ resistor in parallel with meter.
- (d) 50 mΩ resistor in series with meter.

Ans: $I_m = 2A$, $I = 10A$

$I_m R_m = 10 mV$

$R_{SH} = \frac{I_m R_m}{I - I_m} = \frac{10 \times 10^{-3}}{10 - 2} = 1.25 m\Omega$ (option-a)

Q) A Galvanometer with a full scale current of 10mA as a resistance of 1000Ω, The multiplying power of 100Ω shunt with this galvanometer is.

- (a) 110 , (b) 100 , (c) 11 , (d) 10.

Ans:

$m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{SH}} = 1 + \frac{1000}{100} = 11$ (option-c)

Q) The Ammeter shown in the fig; Let the PMMC instrument of coil resistance $R_m = 99\Omega$ and a full scale deflection current of 0.1 mA. The shunt resistor $R_{SH} = 1\Omega$, the total current passing through ammeter at 25% of full scale is,

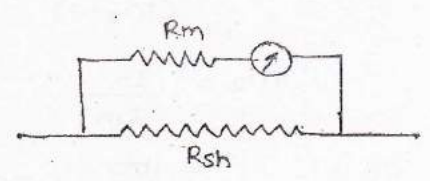
- (a) 0.50 mA , (b) 2.45 mA , (c) 2.5 mA , (d) 2.475 mA.

Ans: $R_m = 99\Omega$, $I_m = 0.1 mA$
 $R_{sh} = 1\Omega$

$m = 1 + \frac{R_m}{R_{sh}} = 1 + \frac{99}{1} = 100$

total current at FSD,

$m = \frac{I}{I_m} \Rightarrow I = m I_m = 100 \times 0.1 = 10 mA$



$\therefore \theta \propto I$
 at 25% of FSD = 25% of current flows through meter
 $\therefore 25\% \text{ of } I = \frac{25}{100} \times 10 \text{ mA} = 2.5 \text{ mA}$ (option-c).

Q) 2 mA ammeter with full scale currents of 1 mA and 10 mA are connected in parallel and they read 0.5 mA and 2.5 mA resp. Their internal resistances are in the ratio of

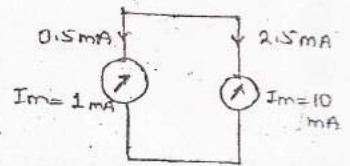
(a) 1:10

(c) 1:5

(b) 10:1

(d) 5:1

Ans $\Rightarrow R_1 = \frac{V_m}{I_1}$, $R_2 = \frac{V_m}{I_2}$



$$\frac{R_1}{R_2} = \frac{V_m}{I_1} \times \frac{I_2}{V_m} = \frac{I_2}{I_1} = \frac{10}{1} = 10:1 \quad (\text{option-b})$$

Q) Design a universal shunt meter with current ranges of 0.1 mA, 10 mA, 50 mA with DC ammeter of internal resistance of 100Ω and full scale current of $100 \mu\text{A}$.

Ans $\Rightarrow R_m = 100 \Omega$, $I_m = 100 \mu\text{A}$, $I_1 = 0.1 \text{ mA}$, $I_2 = 10 \text{ mA}$, $I_3 = 50 \text{ mA}$

As I_m and I_1 corresponds to the same value of current, the meter is used directly for range one (i.e. I_1).

For, $I_2 = 10 \text{ mA}$

$$m_2 = \frac{I_2}{I_m} = \frac{10 \times 10^{-3}}{0.1} = 100 \quad \frac{10 \times 10^{-3}}{100 \times 10^{-6}} = 100$$

$\therefore R_2 = \frac{R_m}{(m_2 - 1)}$ { as I_2 is the first range }

$$m_2 = \frac{I_2}{I_m} = \frac{100}{100 - 1} = 1.01 \Omega$$

Similarly, $R_3 = \frac{R_m + R_1}{m_3}$; $m_3 = \frac{I_3}{I_m}$

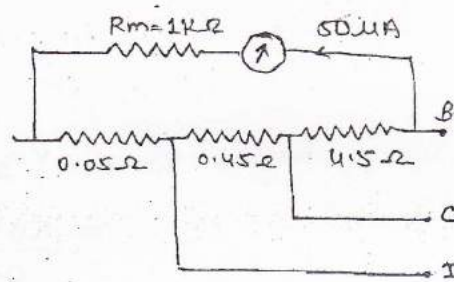
$$m_s = \frac{50 \times 10^{-3}}{100 \times 10^{-6}} = 500$$

$$R_s = \frac{100 + 1.01}{500} = 0.20 \Omega$$

Q) A PMMC instrument has three resistance Agston shunt in shown in figure. The ammeter has a resistance of 1k Ω and FSD of 50 μ A. Calculate the values of the three ranges of the Ammeter.

Ans \Rightarrow Case-I :- Switch at position - B,

voltage drop across meter = $I_m R_m$
 $= 50 \times 10^{-6} \times 10^3$
 $= 50 \text{ mV}$



\therefore As the same voltage is across the shunt, we have hence the current through shunt will be,

$$I_{SH} = \frac{V}{R_{SH}}$$

here, $R_{SH} = 0.05 + 0.45 + 4.5 = 5 \Omega$

$\therefore I_{SH} = \frac{50 \times 10^{-3}}{5} = 10 \times 10^{-3} \text{ A} = 10 \text{ mA}$

Total current = $I_m + I_{SH}$
 $= 50 \times 10^{-6} + 10 \times 10^{-3}$
 $\approx 10 \text{ mA}$

* Case-II :- Switch at position - C,

$R_{sh} = 0.05 + 0.45$

$I_m = 50 \mu\text{A}$

$R_m = 1\text{k} + 4.5 \Omega$

voltage drop across the meter = $I_m R_m$
 $= 50 \times 10^{-6} \times 1004.5$
 $= 50.225 \text{ mV}$

$$\text{Hence, } I_{SH} = V/R_{SH} = \frac{50 \cdot 225 \times 10^{-3}}{0.50} = 100.450 \text{ mA}$$

$$\begin{aligned} \text{Total current} &= I_m + I_{SH} \\ &= 50 \times 10^{-6} + 100.45 \times 10^{-3} \\ &= 100.5 \times 10^{-3} = 100.5 \text{ mA} \end{aligned}$$

* Case-3 :- When switch is at position-D,

$$R_{SH} = 0.05 \Omega ; I_m = 50 \mu\text{A}$$

$$R_m = 1\text{K} + 4.5 \Omega + 0.45 \Omega = 1004.55$$

$$\begin{aligned} \text{voltage drop across the meter} &= I_m R_m \\ &= 50 \times 10^{-6} \times 1004.55 \\ &= 50.2275 \text{ mV} \end{aligned}$$

$$\begin{aligned} \text{Now, } I_{SH} &= \frac{V}{R_{SH}} \\ &= \frac{50.2275}{0.05} \text{ mA} = 1004.55 \text{ mA} \end{aligned}$$

$$\begin{aligned} \text{Total current, } I &= I_{SH} + I_m \\ &= 1004.55 \times 10^{-3} + 50 \times 10^{-6} \\ &= 1004.6 \times 10^{-3} \\ &= 1004.6 \text{ mA} \quad (\text{Ans}) \end{aligned}$$

⇒ PMMC Voltmeter :-

- A Basic PMMC instrument is modified to measure voltage by connecting a high resistance in series with meter movement.
- This high resistance known as the multiplier resistance, limits the current passing through the meter to a small value, thereby protecting the meter from damage.

→ Here,

$$V = I_m (R_s + R_m)$$

$$V = I_m R_s + I_m R_m$$

$$I_m R_s = V - I_m R_m$$

$$R_s = \frac{V}{I_m} - R_m \quad \text{--- (1)}$$

Here the multiplying factor of the multiplier is,

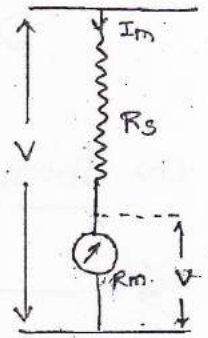
$$m = \frac{V}{V}$$

$$m = \frac{I_m(R_s + R_m)}{I_m R_m}$$

$$m = \frac{R_s}{R_m} + 1 \quad \text{--- (2)}$$

Expressing 'Rs' in terms of m, we get

$$R_s = (m-1)R_m \quad \text{--- (3)}$$



- Temperature related errors in a PMMC voltmeter occur due to the change in resistance of the multiplier resistor due to heating effect of electric current. (As copper form a very small component of the series combination).
- If the multiplier resistor is fabricated with Manganin then it would act as a swamping resistor itself, thereby compensating for the error due to temperature variation.

Multi-range voltmeter :-

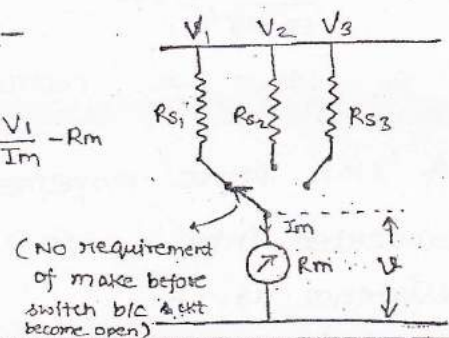
• Two Method :-

(a) Individual Multiplier method :-

$$R_{s1} = (m_1 - 1)R_m \quad ; \quad m_1 = \frac{V_1}{V} \quad ; \quad R_{s1} = \frac{V_1}{I_m} - R_m$$

$$R_{s2} = (m_2 - 1)R_m \quad ; \quad m_2 = \frac{V_2}{V}$$

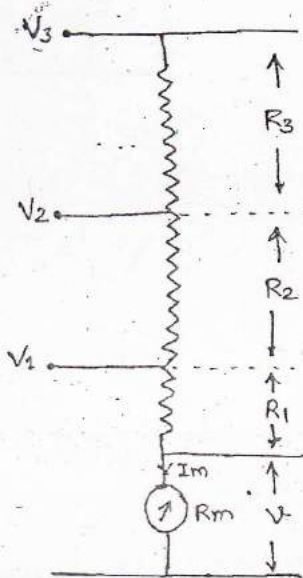
$$= \frac{V_2}{I_m} - R_m$$



$$R_{s3} = (m_3 - 1)R_m \quad ; \quad m_3 = \frac{V_3}{I_m}$$

$$= \frac{V_3}{I_m} - R_m$$

(b) Potential Divider Arrangement :-



Here,

$$\checkmark R_1 = (m_1 - 1)R_m \quad ; \quad m_1 = \frac{V_1}{I_m}$$

$$\checkmark R_2 = (m_2 - 1)R_m - R_1$$

$$= (m_2 - 1)R_m - (m_1 - 1)R_m$$

$$= m_2 R_m - R_m - m_1 R_m + R_m$$

$$\checkmark R_2 = (m_2 - m_1)R_m$$

$$\checkmark R_3 = (m_3 - m_2)R_m$$

Q) A basic PMMC movement with FSD of current of $50 \mu A$ and internal resistance of 500Ω is used as a voltmeter. The multiplier resistance needed to extend the range of instrument to measure range of $10 V$ is given by :-

(a) $100 K \Omega$

(c) $199.5 K \Omega$

(b) $500 K \Omega$

(d) $2 \times 10^{-5} K \Omega$

Ans $\Rightarrow I_m = 50 \mu A$, $R_m = 500 \Omega$, $V = 10 V$

$$R_s = \frac{V}{I_m} - R_m$$

$$= \frac{10}{50 \times 10^{-6}} - 500$$

$$R_s = 199.5 K \Omega \quad (\text{option - c})$$

Q) A $1 mA$ PMMC movement has a resistance of 100Ω . It is to be converted into a $10 V$ voltmeter, the value of its multiplier resistance is :-

(a) 999 Ω

(b) 9999 Ω

(c) 9900 Ω

(d) 990 Ω

Ans $\Rightarrow I_m = 1\text{mA}$, $R_m = 100\Omega$, $V = 10\text{V}$.

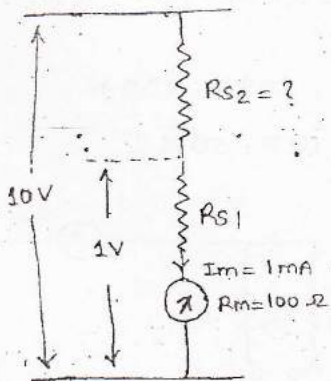
$$R_s = \frac{V}{I_m} - R_m$$

$$= \frac{10}{1 \times 10^{-3}} - 100 = 10000 - 100 = 9900 \Omega \text{ (option-c)}$$

Q) A Galvanometer with a internal resistance of 100Ω has a FSD of 1mA . If it is used to realise a DC voltmeter with a full scale range of 1V . The value of external resistance required to extend the range of this voltmeter to 10V is -

Ans $\Rightarrow R_m = 100\Omega$

(a) 900 Ω (b) 9000 Ω
(c) 9900 Ω (d) None



for 1V range,

$$R_m = 100\Omega$$

$$I_m = 1\text{mA}$$

$$V = 1\text{V}$$

$$R_{s1} = \frac{V}{I_m} - R_m$$

$$= \frac{1}{1 \times 10^{-3}} - 100 = 900\Omega$$

for 10V range,

$$V = 10\text{V}, v = 1\text{V}, R_m = 900 + 100 = 1000\Omega$$

$$R_{s2} = (m-1)R_m$$

$$m = \frac{V}{v} = \frac{10}{1} = 10$$

$$\therefore R_{s2} = (10-1) \times 1000 = 9000\Omega \text{ (option-b)}$$

Q) A moving coil meter has a internal resistance of 10Ω and FSD current of 5mA . The meter is used to measure current of 1A by adding a shunt resistor R_{SH} . The same meter is converted into voltmeter of 0 to 5V scale by adding a series resistance ' R_s ', then value of resistances are -

(a) $R_{SH} = 1000 \Omega$, $R_s = 0.05 \Omega$

(b) $R_{SH} = 0.05 \Omega$, $R_s = 990 \Omega$

(c) $R_{SH} = 0.05 \Omega$, $R_s = 1000 \Omega$

(d) None of the above.

Ans \Rightarrow For ammeter,

$$R_{SH} = \frac{I_m R_m}{I - I_m}$$

$$= \frac{5 \times 10^{-3} \times 10}{1 - 5 \times 10^{-3}} = 0.05 \Omega$$

For voltmeter,

$$R_s = \frac{V}{I_m} - R_m$$

$$= \frac{5}{5 \times 10^{-3}} - 10 = 990 \Omega \quad \text{(Option-b)}$$

Q) A multi-range voltmeter shown in fig. with ASD of current of $50 \mu A$ and a meter resistance of $1 K \Omega$. What are the values of resistances R_1 , R_2 , R_3 resp.

(a) $100 K$, $200 K$, $1000 K$.

(c) $99 K$, $109 K$, $199 K$.

(b) $101 K$, $201 K$, $1001 K$.

(d). $5 K$, $10 K$, $50 K$.

Ans \Rightarrow $I_m = 50 \mu A$, $R_m = 1 K \Omega$

$$R_{s1} = \frac{V_1}{I_m} - R_m$$

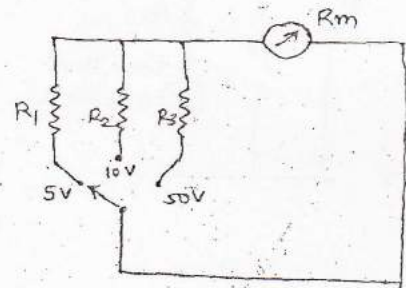
$$= \frac{5}{50 \times 10^{-6}} - 1000 = 99 K$$

$$R_{s2} = \frac{V_2}{I_m} - R_m$$

$$= \frac{10}{50 \times 10^{-6}} - 1000 = 109 K$$

$$R_{s3} = \frac{V_3}{I_m} - R_m$$

$$= \frac{50}{50 \times 10^{-6}} - 1000 = 199 K \quad \text{(Option-c)}$$



Q) A PMMC instrument has ASD current of $50 \mu A$ and meter resistance of 500Ω is to be converted into voltmeter with the ranges of $10 V$, $50 V$, $100 V$. When the switch is connected to position A, B and C resp. as shown in figure, what are the

values of resistance R_1 , R_2 and R_3 resp.

Ans: $I_m = 50 \mu A$

$R_m = 1700 \Omega$

$$R_1 = \frac{V_1}{I_m} - R_m$$

$$= \frac{10}{50 \times 10^{-6}} - 1700$$

$$= 198.3 \text{ K}\Omega$$

$$R_2 = \left(\frac{V_2}{I_m} - R_m \right) - R_1$$

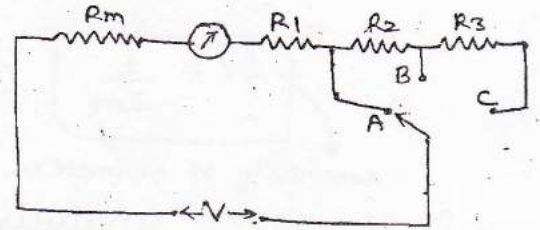
$$= \left(\frac{50}{50 \times 10^{-6}} - 1700 \right) - 198.3 \text{ K}\Omega$$

$$= 800 \text{ K}\Omega$$

$$R_3 = \left(\frac{V_3}{I_m} - R_m \right) - (R_1 + R_2)$$

$$= \left(\frac{100}{50 \times 10^{-6}} - 1700 \right) - (198.3 + 800)$$

$$= 1 \text{ M}\Omega \quad \text{Ans}$$



Voltmeter Sensitivity :-

The sensitivity of any instrument is defined as 'the minimum value of I/P that instrument can sense', alternatively 'sensitivity is the rate of change of o/p w.r.t I/p'.

Mathematically expressed sensitivity is equal to output by Input.

The sensitivity of voltmeter is defined as the resistance offered by the voltmeter per volt of deflecting voltage.

$$S_v = \frac{R_T}{V} \quad (\Omega/V)$$

$$R_T = R_s + R_m$$

$$V = I_m (R_s + R_m)$$

$$S_v = \frac{(R_s + R_m)}{I_m (R_s + R_m)}$$

$$S_v = \frac{1}{I_m} \quad (\Omega/V)$$

sensitivity of voltmeter.

- From above analysis it can be seen that the sensitivity actually represent the current required by the voltmeter to produce full scale deflection.
- ✓ A voltmeter is said to be highly sensitive, if it requires a very small value of current to produce FSD.
- ✓ Low sensitive voltmeter draw a large value of current from the ckt in which they are introduced, thereby causing the errors due to the loading effect of the voltmeter.

→ we know,

$$R_s = \frac{V}{I_m} - R_m$$

$$R_s = S_v \cdot V - R_m$$

$$R_s = S_v (V - V)$$

↳ The sensitivity figure can also be used to calculate the value of multiplier resistance (R_s) as shown above.

Q. Which of the voltmeter is more sensitive and why?

(1) voltmeter-A having a range of (0 to 10)V with a multiplier resistance of 18 K Ω .

(2) voltmeter-B having a range of (0 to 300)V with a multiplier resistance of 298 K.

It is each of the meter has an internal resistance of 2K Ω .

Ans → Since, $R_s = S_v \cdot V - R_m$.

$$\text{So, } S_v = \frac{R_s + R_m}{V} \quad \& \quad S_v = \frac{1}{I_m}$$

For voltmeter-A :-

$$S_v = \frac{18+2}{10} = 2$$

$$I_m = \frac{1}{2} = 0.5 \text{ mA}$$

For voltmeter-B :-

$$S_v = \frac{298+2}{300} = 1$$

$$I_m = \frac{1}{1} = 1 \text{ mA}$$

→ From the above analysis, it can be seen that voltmeter-A has more sensitive than voltmeter-B, as it requires half the current as required in voltmeter-B to produce full scale deflection.

Q) The sensitivity of a 200 μA meter movement when it is used as DC voltmeter is given by,

(a) $500 \Omega/\text{V}$

(c) $0.5 \Omega/\text{V}$

(b) $5 \Omega/\text{V}$

(d) $5 \Omega/\text{mV}$

Ans → $S_v = \frac{1}{I_m} = \frac{1}{200 \times 10^{-6}} = 0.5 \times 10^4 = 5000 \Omega/\text{V}$ (option-d)

Q) What is the series resistance needed to extend the range of a (0 to 100) V voltmeter, with sensitivity of $200 \text{ k}\Omega/\text{V}$ to (0 to 1000) V.

(a) $1 \text{ M}\Omega$

(b) $16 \text{ M}\Omega$

(c) $18 \text{ M}\Omega$

(d) $20 \text{ M}\Omega$

Ans → Since, $R_s = S_v (V - v)$

$$\therefore V = 1000 \text{ V}, v = 100, S_v = 200 \text{ k}\Omega/\text{V}$$

$$\text{So, } R_s = 200 \times 10^3 (1000 - 100)$$

$$R_s = 18 \text{ M}\Omega \quad (\text{option-c})$$

Q) In ckt shown in fig, voltage across R_2 is measured by two different voltmeter P and Q having sensitivity of $1 \text{ k}\Omega$ and $20 \text{ k}\Omega/\text{V}$ resp.. Both the meter are used on the 50V scale which of the following statements are true.

(a) P is more accurate than Q.

(b) Both meters read same voltage.

(c) Q is more accurate than P.

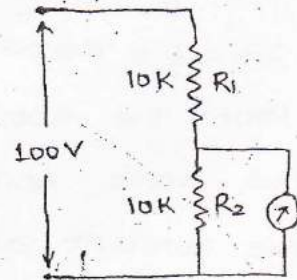
(d) None of the above.

Ans → (option-c) (since Q required less current than P for FSD).

For P:- $S_v = \frac{1}{I_m}$

$$I_m = \frac{1}{S_v} = \frac{1}{1 \text{ k}\Omega/\text{V}} = 1 \text{ mA}$$

For Q:- $S_v = \frac{1}{I_m} \Rightarrow I_m = \frac{1}{20 \text{ k}\Omega/\text{V}} = 0.05 \text{ mA}$



Q) Three DC voltmeter are connected in series across a 120V DC supply. These voltmeters are specified as :-

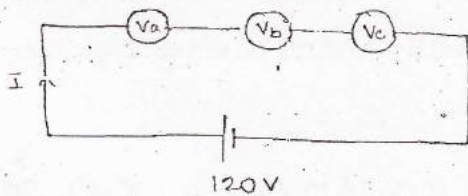
voltmeter A : 100V, 5mA

voltmeter B : 100V, 250 Ω/V .

voltmeter C : 10mA, 15000 Ω .

The voltages read by the individual meters are :-

Ans →



Here, $R_a = \frac{100}{5 \text{ mA}} = 20000 \Omega$

$$R_b = 100 \times 250 = 25000 \Omega$$

$$R_c = 15000 \Omega$$

since, voltages are in series \therefore their resistances are also in series.

$$R_T = R_a + R_b + R_c$$

$$= 60 \text{ k}\Omega$$

$$\text{Now, } I = \frac{V}{R_T} = \frac{120 \times 10^{-3}}{60} = 2 \times 10^{-3} \text{ A}$$

$$V_a = I R_a = 2 \times 10^{-3} \times 20 \times 10^3 = 40 \text{ V}$$

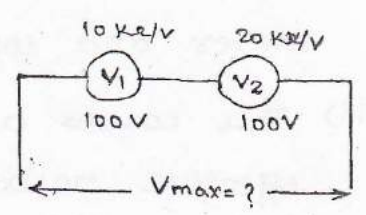
$$V_b = I R_b = 2 \times 10^{-3} \times 25 \times 10^3 = 50 \text{ V}$$

$$V_c = I R_c = 2 \times 10^{-3} \times 15 \times 10^3 = 30 \text{ V} \quad (\text{Ans})$$

Q) Two 100V full scale PMMC type DC voltmeter having figure of merits of 10 $\text{k}\Omega/\text{V}$ and 20 $\text{k}\Omega/\text{V}$ are connected in series. This series combination can be used to measure a maximum DC voltage of

Volts.

Ans) From the sensitivity figure it can be seen that the voltmeter V_2 requires half the current requires by V_1 to produce ASD.



↳ The max. current that can be flows through the series combination without damaging either of meters is ASD of current of V_2 .

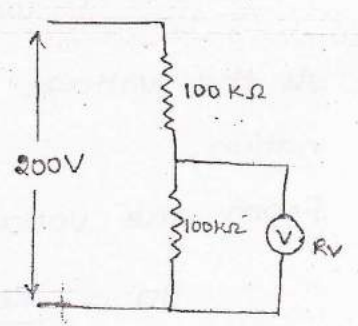
↳ As $\theta \propto V$, hence when the ASD current of V_2 flows through the combination, V_2 would indicates 100V while V_1 indicates 50V.

↳ Thus the max. voltage that series combination can measure is $100 + 50 = 150 \text{ V}$ (Ans)

⇒ Voltmeter Loading :-

Q) When a voltmeter connected across either of the ^{two 100kΩ} ~~resistors~~ shown in figure, it shows the reading of 90V when it should have show 100V. Explain clearly why this happening, also find the internal resistance of voltmeter.

Ans → (i) A voltmeter is connected across any one of the two 100kΩ resistance would act as shunt to that part of ckt, thereby reducing the effective resistance of parallel combination.



(ii) The effective resistance of parallel combination in this

Case would be lower than the resistance of both voltmeter and $100\text{ k}\Omega$ resistance.

- (iii) This causes a larger current to be drawn by the effective resistances resulting in a lower voltages.
- (iv) The voltmeter actually reads the voltage across the parallel combination rather than the voltage across the $100\text{ k}\Omega$ resistance, thereby introducing error in the measurement of voltages.
- (v) These errors are known as the error due to loading effect of the voltmeter and significantly large in instances where :-
- (a) A low sensitive voltmeter is used in voltage measurement.
 - (b) voltage is measured in a part of a highly resistive circuit.

→ Numerical part :-

The effective resistance across the parallel combination is
($R_v \parallel 100\text{ k}\Omega$)

$$\text{So, } R_e = \frac{100 R_v}{100 + R_v}$$

As the voltmeter reads the voltage across the parallel combination,

from the voltage divider we have

$$90 = \frac{R_e \times 200}{R_e + 100}$$

$$90(R_e + 100) = 200 R_e$$

$$90 R_e + 9000 = 200 R_e$$

$$110 R_e = 9000$$

$$R_e = \frac{9000}{110} = 81.81 \text{ K}\Omega$$

$$\therefore 81.81 = \frac{R_v \times 100}{(R_v + 100)}$$

$$R_v(100 - 81.81) = 8181$$

$$R_v = 449.75 \text{ K}\Omega \quad (\text{Ans})$$

Q) The voltmeter shown in the figure, as a sensitivity of $500 \Omega/\text{V}$. FSD voltage of 100V . Calculate the value of R_x when the voltmeter is connected as shown in the ckt and the meter reads 20V .

Ans \rightarrow Resistance of voltmeter,

$$= S_v \cdot V$$

$$= 500 \times 100$$

$$= 50 \text{ K}\Omega$$

As the voltmeter reads voltage across the parallel combination ($R_x \parallel 50\text{K}$), the effective resistance will be :-

$$R_e = \frac{R_x \times 50}{R_x + 50}$$

As the voltmeter reads 20V .

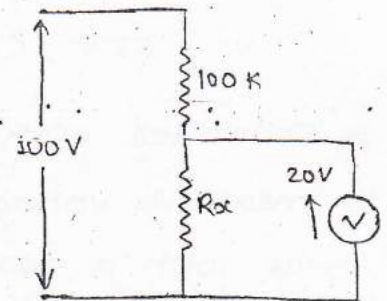
\therefore From the voltage divider,

$$20 = \frac{R_e \times 100}{R_e + 100}$$

$$R_e = 25 \text{ K}\Omega$$

\therefore The value of R_x is :

$$25 = \frac{R_x \times 50}{R_x + 50} \Rightarrow R_x = 50 \text{ K}\Omega \quad (\text{Ans})$$



Q) A voltmeter connected across a $10\text{K}\Omega$ resistor as shown in fig. reads 5V . The voltmeter is rated as $1000\Omega/\text{V}$ and as a FSD reading of 10V . Calculate the value of supply voltage V_s .

Ans Resistance of voltmeter is,

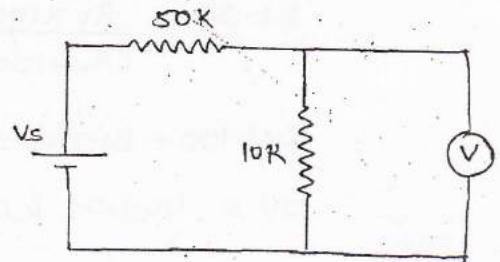
$$\begin{aligned} &= S_V \cdot V \\ &= 1000 \times 10 \\ &= 10\text{K}\Omega. \end{aligned}$$

As voltmeter reads the voltage across parallel combination of $(10\text{K} \parallel 10\text{K}\Omega) = 5\text{K}\Omega$.

From voltage division rule,

$$5\text{V} = \frac{5 \times V_s}{5 + 50}$$

$$V_s = 55\text{V} \quad (\text{Ans})$$



Q) A voltmeter with an internal resistance of 4750Ω is used to measure voltage across a resistor of 600Ω connected in series with a resistance of 400Ω . What is the % error in the measurement.

Ans effective resistance of the parallel combination,

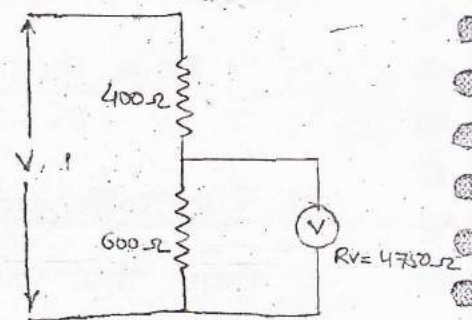
$$R_e = \frac{4750 \times 600}{4750 + 600} = 532.7\Omega$$

Now, measured value is the reading of the voltmeter which reads the voltage across R_e .

$$MV = \frac{532.7\text{V}}{532.7 + 400} = 0.57\text{V}$$

True value is voltage across 600Ω

$$TV = \frac{600 \times V}{600 + 400} = 0.6\text{V}$$



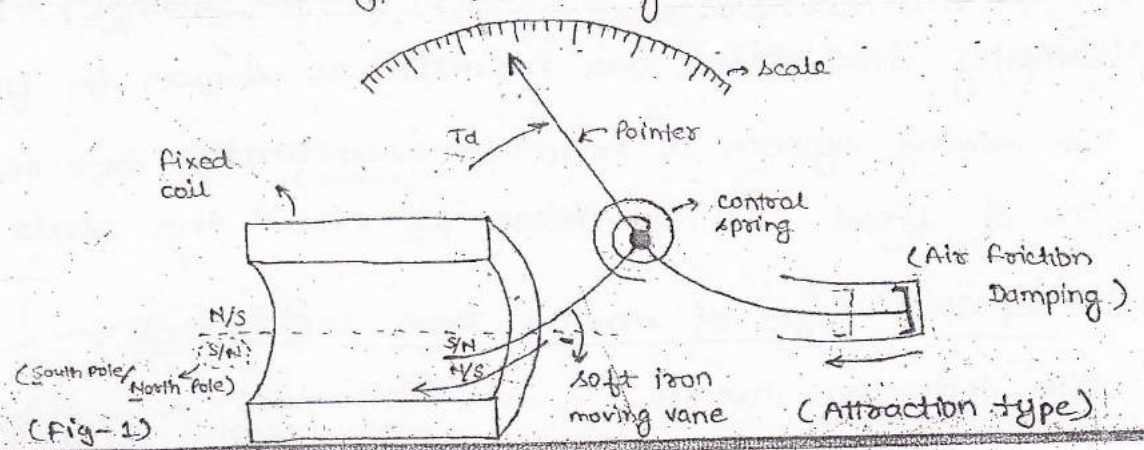
$$\begin{aligned} \% \text{ error} &= \frac{MV - TV}{TV} \times 100 \\ &= \left(\frac{0.57V - 0.6V}{0.6V} \right) \times 100 \\ &= -5\% \quad (\text{Ans}) \end{aligned}$$

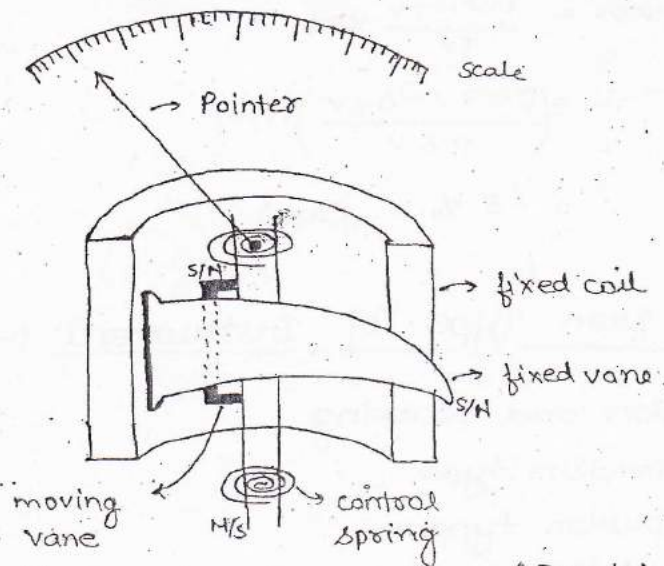
⊙ Moving Iron Type of Instrument :- (It can measure both AC and DC)

- ↳ construction and working.
 - Attraction type.
 - Repulsion type.
- ↳ Advantages and disadvantages.
- ↳ Sources of error.
- ↳ Derivation for T_d .
- ↳ Application.

Construction & working :-

- Moving Iron type of instrument is based on the principle on the magnetic effect of electric current.
- Depending on how they produce a deflecting torque, the instrument are further classified as :-
 - (i) Attraction type of moving instrument.
 - (ii) Repulsion type of moving instrument.





(Fig-2)

(Repulsion Type)

(i) Attraction type of moving Iron Instrument :-

- The deflecting torque in this instrument is produced by a force of attraction b/w a fixed and m acting on a moving vane due to magnetic field of opposite polarity induced on the surface by the magnetic field of fixed coil.
- The fixed system of this instrument consist of a 'C' shape insulating former with a thick copper wire wound on the surface. known as the fixed coil instrument, it carries current under measurement.
- The moving system of the instrument is consist of a spindle on to which a pointer, a set of control springs, an air friction damping mechanism are mounted as shown in figure-1.
- The moving system is mounted ^(external) eccentrically w.r.t the central axis of fixed coil in order to extend the scale span.

(ii) Repulsion type of moving Iron Instrument :-

- The deflecting torque in this instrument is produced by a

force of repulsion b/w the fixed and the moving span due to similar magnetic polarity induced on their surface by the magnetic field of the fixed coil.

- The fixed system of this instrument is consist of a fixed coil that carries the current under measurement.
- A soft iron vane is non-magnetically cemented to the internal surface of a fixed coil as shown in fig-2. The moving system of this instrument consist of a spindle on to which a set of control springs, a pointer, an air friction damping mechanism and a soft iron moving vane are mounted as shown.
- The moving system of this instrument is mounted concentrically wot the central axis of the fixed coil as in order extend the scale span.

⇒ Points common for both construction :-

- (i) The controlling torque in this instrument is produced by a spring control mechanism and due to presence of weak magnetic field and an air friction damping mechanism is used to produce damping torque.
- (ii) The expression that govern the deflecting torque produce in this instrument is given by:-

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \quad (\text{Nm})$$

As a spring control is used,

$$T_c = K\theta$$

at steady state position we have,

$$\theta \propto I^2 \frac{dL}{d\theta}$$

⇒ Advantage :-

- As the direction of the magnetic field changes with the change in the polarity of the AC parameters under measurement, this instrument can be used for both AC and DC parameters.
- As the current under measurement is passed through a fixed wire which is wound by the thick wire, this instrument has a large current capacity.

Note :- Moving Iron type instrument can directly used as a Ammeter upto a range of 50 A without a use of a shunt.

- As instrument exhibit a square law response, the angular deflection of an instrument is directly in terms of RMS value of the AC parameter under measurement.

⇒ Disadvantage :-

- As the compensation required for both AC and DC are different, these instrument will have different calibration for both AC and DC parameters.

Note :- An instrument calibrated on AC if used on DC will be over compensated for error and hence gives an higher reading and vice versa is also true.

- Due to the presence of weak field due to the fixed coil, these instruments are affected by stray magnetic field.

• As $\theta \propto I^2$, these instrument give a non-uniform scale.

⇒ Sources of errors :-

(1) Common for both AC and DC :-

(a) Error due to ageing of spring.

(b) Error due to the change in resistance of the fixed coil b/c of the heating effect of electric current.

(2) Error specific for AC :-

(a) Error due to Eddy current, can be compensated by using a thick multi-stranded wire to wind the fixed coil.

(b) Error due to Hysteresis, can be compensated by replacing the soft iron vane with a vane made up of nickel-iron alloy and the errors are further compensated by reducing the surface area of vane.

(c) Error due to frequency, the error due to frequency which occur is because of the change in impedance of fixed coil due to the variation in frequency can be compensated by introducing a proportionate capacitive reactance in a fixed coil circuit.

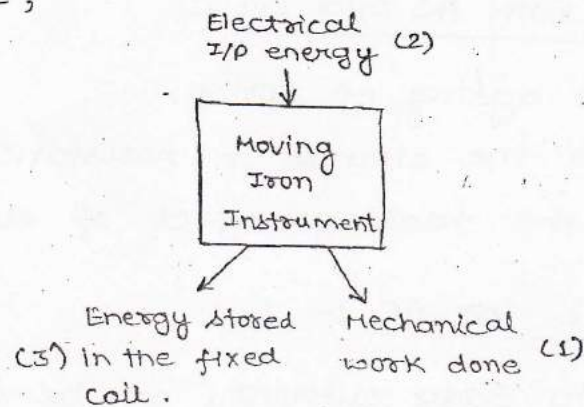
↳ Error due to frequency is because of Harmonic content are neglected in this instrument.

⇒ Derivation for generalized expression of the deflecting torque :-

• The mathematical analysis of the deflecting torque is based on the law of conservation of energy which states that

energy is neither created nor destroyed, it only changes from one form to other.

In this case,



The expression is derived on the basis of the energy relation shown in above figure.

Initial condition,

Initial current = I

Angular deflection = θ

self Inductance = L .

If an incremental current is supplied to a system, the angular deflection changes by $d\theta$ and some mechanical work is done.

If ' T_d ' is the deflecting torque.

then mechanical work done,

$$= T_d \cdot d\theta \quad \dots \dots (1)$$

As I changes by dI

θ changes by $d\theta$

& L changes by dL

A proportional change also take place in the EMF expressed

as,

$$e = \frac{d}{dt}(LI) = L \frac{dI}{dt} + I \frac{dL}{dt}$$

MEASUREMENT

The electrical input energy will be,

$$e I dt = I dt \left\{ L \frac{dI}{dt} + I \frac{dL}{dt} \right\}$$

∴ electrical I/p energy,

$$I L dI + I^2 dL \dots\dots (2)$$

the initial energy stored in the fixed coil is,

$$= \frac{1}{2} I^2 L$$

This energy changes to,

$$= \frac{1}{2} (I + dI)^2 (L + dL)$$

∴ change in the energy stored is,

$$= \frac{1}{2} (I + dI)^2 (L + dL) - \frac{1}{2} I^2 L$$

simplifying and neglecting the higher order terms, then change in energy stored,

$$= I L dI + \frac{1}{2} I^2 dL \dots\dots (3)$$

from the law of conservation of energy stated about us have,

Electrical I/p energy = change in energy stored + Mechanical work done.

$$\Rightarrow I L dI + I^2 dL = I L dI + \frac{1}{2} I^2 dL + T_d \cdot d\theta$$

$$\Rightarrow T_d \cdot d\theta = \frac{1}{2} I^2 dL$$

$$\Rightarrow \left\{ T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \right\} \text{ (Nm)} \dots\dots (4)$$

As spring control is used,

$$T_c = k\theta$$

at steady state position, $T_c = T_d$.

$$K\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\left\{ \theta = \frac{1}{2} \frac{I^2}{K} \frac{dL}{d\theta} \right\} \text{ (rad)} \quad \dots (5)$$

from expression (5) it can be said that

$$\theta \propto I^2 \frac{dL}{d\theta}$$

Hence the instrument will exhibit a square law response, therefore :

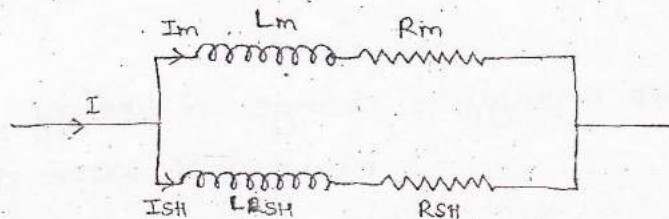
(a) Its angular deflection is directly in terms of the RMS value of the AC parameter.

(b) The instrument gives a non-uniform scale.

⇒ Application of MI Instrument :- (Moving Iron)

(1) MI Ammeter :-

In order to measure current beyond 50 A in a MI instrument, a low resistance is connected across the circuit.



$$\text{Here, } R_{sh} = \frac{R_m}{(m-1)} \quad ; \quad m = \frac{I}{I_m}$$

In order to make the above instrument suitable to measure currents at all frequency, the time constant of the shunt and meter are made equal.

$$\frac{L_{sh}}{R_{sh}} = \frac{L_m}{R_m} \Rightarrow \boxed{L_{sh} = \frac{L_m \cdot R_{sh}}{R_m}}$$

(2) MI Voltmeter :-

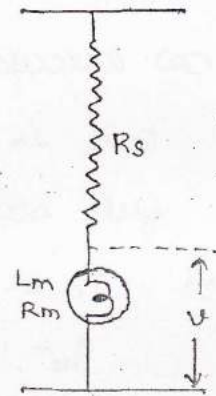
- A moving Iron instrument is modified to measure voltage by connecting a high non-inductive resistance in series with a meter movement.

$$R_s = (m-1) R_m \quad ; \quad m = \frac{V}{V}$$

Here,

$$m = \frac{I_m (R_s + R_m + j\omega L_m)}{I_m (R_m + j\omega L_m)}$$

$$\left\{ m = \sqrt{\frac{(R_s + R_m)^2 + (\omega L_m)^2}{R_m^2 + (\omega L_m)^2}} \right\}$$



Q The inductance of a moving Iron Instrument is given by a expression, $L = (30 + 10\theta - 2\theta^2) \mu\text{H}$ where θ is deflections in radians. If the control spring constant of the Instrument is $25 \times 10^{-6} \text{ Nm/rad}$. Calculate the value of the deflection for a current of 5A.

Ans $\rightarrow L = (30 + 10\theta - 2\theta^2) \mu\text{H}$

$$K = 25 \times 10^{-6} \text{ Nm/rad}$$

$$I = 5\text{A}$$

Here, $\frac{dL}{d\theta} = (10 - 4\theta) \times 10^{-6} \text{ H/rad}$

we know,

$$\theta = \frac{1}{2} \frac{I^2}{K} \cdot \frac{dL}{d\theta}$$

$$= \frac{1}{2} \cdot \frac{(5)^2}{25 \times 10^{-6}} \times (10 - 4\theta) \times 10^{-6}$$

$$2\theta = 10 - 4\theta$$

$$6\theta = 10$$

$$\theta = 1.66 \text{ radian (Ans)}$$

Q) A moving iron ammeter produces a full scale ~~for~~ torque of $240 \mu\text{Nm}$ with the deflection of 120° at a current of 10 A .

(a) Calculate the rate of change of self inductance at the full scale.

(b) Calculate the ~~rate~~. If the initial inductance of the coil is $8.53 \mu\text{H}$. The final inductance of the coil at full scale will be :-

Ans $\Rightarrow T_d = 240 \mu\text{Nm}$.

$$\theta_{\text{FSD}} = 120^\circ = 2\pi/3$$

$$L_{\text{initial}} = 8.53 \mu\text{H}$$

$$I = 10 \text{ A}$$

(a) we know,

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\left\{ \frac{dL}{d\theta} = \frac{2T_d}{I^2} \right\}$$

$$\frac{dL}{d\theta} = \frac{2 \times 240 \times 10^{-6}}{(10)^2} = 4.8 \mu\text{H/rad}$$

(b) The change in the inductance of the coil at full scale

$$\text{is, } = \frac{dL}{d\theta} \times \theta_{\text{FSD}}$$

$$= 4.8 \times 10^{-6} \times 2\pi/3$$

$$= 10.05 \mu\text{H}$$

Inductance of coil at full scale deflection,

$$= \text{Initial inductance} + \text{change in Inductance at FSD}$$

$$= 8.53 + 10.05$$

$$= 18.58 \mu\text{H} \quad (\text{Ans})$$

Q Calculate the constants of the shunt to extend the range of a (0 to 5) A MI ammeter to (0 to 50) A. MI instrument constants are $R = 0.09 \Omega$ and $L = 90 \mu\text{H}$.

Ans $I_m = 5 \text{ A}$, $I = 50 \text{ A}$

$R_m = 0.09 \Omega$, $L = 90 \mu\text{H}$

$$R_{SH} = \frac{R_m}{(m-1)} \quad ; \quad m = \frac{I}{I_m} = \frac{50}{5} = 10$$

$$\therefore R_{SH} = \frac{0.09}{(10-1)} = 0.01 \Omega$$

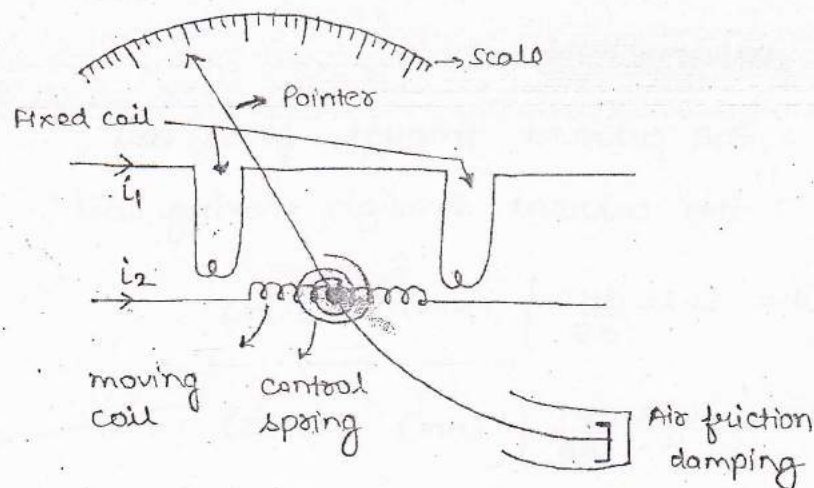
NOW,

$$L_{SH} = \frac{L_m}{R_m} \times R_{SH}$$

$$= \frac{90 \times 10^{-6}}{0.09} \times 0.01 = 10 \mu\text{H} \quad (\text{Ans})$$

Q Electro-dynamo Meter Instrument :-

- An electro-dynamo instrument basis its operation on the magnetic effect of electric current.
- The deflecting torque in this instrument is produced due to the interaction of the current passes through the fixed and the moving coil.



- The fixed system of this instrument consist of the fixed coil

wound with a thick multi-standard wire for a small no. of turns.

- This coil is Air core and split into two parts in order to accomodate the moving system.
- The moving system of the instrument consist of a spindle on to which a pointer, a set of control spring, an air friction damping mechanism and a moving coil wound for a large no. of turns is mounted.
- The moving coil is wound with a thin and light wire and it place b/w two part of fixed coil.
- The controlling torque in this instrument is produced by a spring control mechanism and due to presence of weak operating field, an ~~air~~ friction damping mechanism, is used to produce the damping torque.
- The instantaneous value of deflecting torque is produced in this instrument is given by the expression:

$$\left\{ T_d = i_1 + i_2 \frac{dM}{d\theta} \right\} \text{ ---- (1)} \quad [M \rightarrow \text{Mutual Inductance}]$$

(I) for DC parameters :-

$i_1 = I_1$: the current through fixed coil.

$i_2 = I_2$: the current through moving coil.

$$\left\{ T_d = I_1 \cdot I_2 \frac{dM}{d\theta} \right\} \text{ (Nm) ---- (2)}$$

$$\left\{ \theta = \frac{I_1 \cdot I_2}{K} \frac{dM}{d\theta} \right\} \text{ (Nm) ---- (3)}$$

(II) For AC parameter :-

The average value of the torque over one cycle will be :-

$$T_{av} = \frac{1}{T} \int_0^T T_i dt$$

$$T_{av} = \frac{1}{T} \cdot \frac{dM}{d\theta} \int_0^T i_1 \cdot i_2 dt \quad \dots (4)$$

* (III) For Sinusoidal current :-

Let ϕ be the phase difference b/w i_1 and i_2 .

$$i_1 = I_{m1} \sin(\omega t)$$

$$i_2 = I_{m2} \sin(\omega t - \phi)$$

Substituting in (4) we have,

$$T_{av} = \frac{1}{T} \cdot \frac{dM}{d\theta} \int_0^T I_{m1} \sin(\omega t) \cdot I_{m2} \sin(\omega t - \phi) d(\omega t) \quad [T=2\pi]$$

$$T_{av} = \frac{1}{2\pi} I_{m1} \cdot I_{m2} \cdot \frac{dM}{d\theta} \int_0^{2\pi} \sin(\omega t) \cdot \sin(\omega t - \phi) d(\omega t)$$

$$T_{av} = \frac{I_{m1} \cdot I_{m2}}{2} \cdot \cos \phi \frac{dM}{d\theta}$$

$$T_{av} = \frac{I_{m1}}{\sqrt{2}} \cdot \frac{I_{m2}}{\sqrt{2}} \cdot \cos \phi \cdot \frac{dM}{d\theta}$$

$$\therefore \left\{ T_d = I_1 I_2 \cos \angle_{I_1 I_2} \frac{dM}{d\theta} \right\} \text{ (Nm)}$$

(Cosine of angle b/w I_1 and I_2)

⇒ Advantage :-

- (i) This instrument gives a precision grade accuracy upto a frequency of 10 KHz.

Note :- Precision of an instrument it is ability to consistently gives a same reading when a number of measurement

are made with the same input.

(ii) As a compensation required for both AC and DC are same, these instrument ~~require~~ have same calibration for both AC and DC and hence used as transfer type instrument.

Note :-

- The most common application of an electro-dynamometer ammeter and voltmeters are as transfer type of instrument for calibrating AC ammeters and voltmeter.
 - Thermo-couple type of instrument are more commonly used as transfer type of instrument for the standardization of AC potentiometer.
- *. Depending on how the fixed and moving coils are connected these instrument can be used as ammeter, voltmeter, wattmeter ($V I \cos \phi$), Barimeter ($V I \sin \phi$), frequency meter and power factor meter.

Note :-

- An electro-dynamo meter type power factor meter is characterised by the absence of a control mechanism in its design.
- (iii) As the instrument exhibit square law response, its angular deflection is directly proportional in terms of the RMS value of the AC parameter under measurement.

⇒ Disadvantage :-

- (1) due to low torque to weight ratio, these instruments have

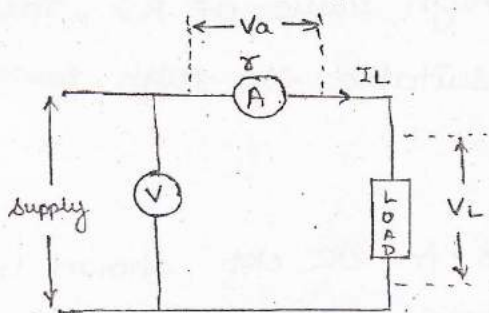
a low sensitivity.

(ii) Due to presence of weak magnetic field, these instrument is easily affected by stray magnetic field.

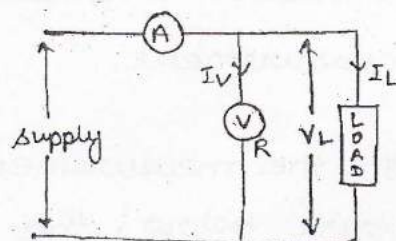
** ⊙ Measurement of Power :-

(I) Power in DC circuit :-

- Power in a DC ckt is expressed as 'the product of the current and the voltage'. Thus a simple voltmeter and ammeter combination could be the simplest and most efficient way for measuring power in a DC ckt.
- The two methodology in which the voltmeter and ammeter are connected to measure power in a DC ckt are, shown below :-



(Fig-1)



(Fig-2)

↪ Fig-1 :-

Calculated power = Ammeter reading × Voltmeter reading

$$= I_L (V_a + V_L)$$

$$= I_L (I_L \cdot r + V_L)$$

$$= I_L^2 \cdot r + I_L V_L$$

$$\% \text{ Error} = \frac{I_L^2 \cdot r}{I_L \cdot V_L} \times 100$$

→ Fig-2 :-

$$\begin{aligned} \text{Calculated power} &= \text{voltmeter reading} \times \text{Ammeter reading} \\ &= V_L \times (I_V + I_L) \\ &= V_L \cdot \left(\frac{V_L}{R} + I_L \right) \\ &= \frac{V_L^2}{R} + V_L I_L \end{aligned}$$

$$\% \text{ Error} = \frac{V_L^2/R}{V_L \cdot I_L} \times 100$$

Note:-

- As the % error in Fig-1, decreases for lower value of the current drawn by the load, this connection methodology is suitable for low power measurement.
- As the % error in Fig-2 decreases for larger value of the load current (due to high value of R), this connection methodology is suitable for high power measurement.

Q) In the measurement of power in DC ckt shown in the figure below, the ammeter and the voltmeter have 0.25Ω and 1750Ω resistances resp. If the voltmeter is connected to the supply side, the measured power compared to True power will be :-

(a) 1.7% less

(c) 0.84% less.

(b) 1.7% more

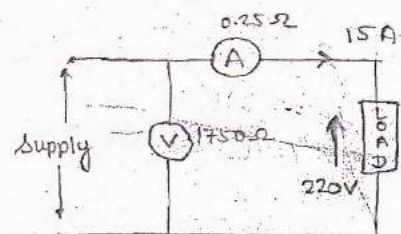
(d) 0.84% more.

$$\begin{aligned} \text{Ans} \Rightarrow \% \text{ Error} &= \frac{I^2 r}{VI} \times 100 \\ &= \frac{15 \times 15 \times 0.25}{220 \times 15} \times 100 \end{aligned}$$

(option-b)

$$= +1.7\%$$

→ +ve represent more (Ans)

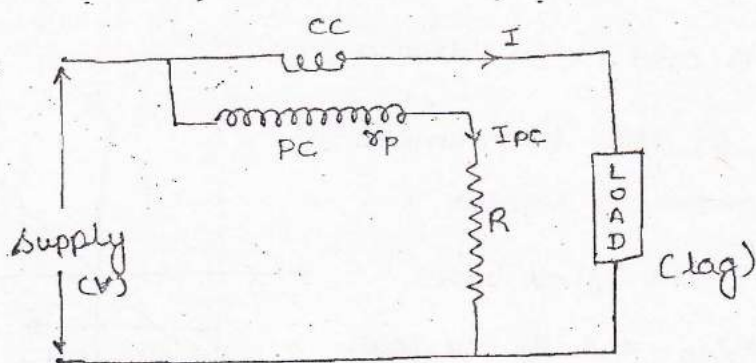


(II) Power in AC circuit (Electro-dynamo wattmeter) :-

- Power in a AC ckt is expressed as the product of current voltage and power factor.
- Thus a simple voltmeter-ammeter combination would be unsuitable to measure power in an AC circuit.
- An electro-dynamo instrument is modified to measure power in AC ckt as follows :-

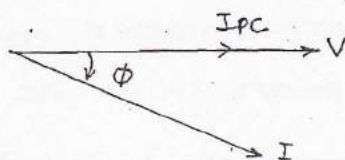
⇒ Construction and working :-

- The fixed coil of the instrument which is wound with thick multi-standard wire is connected in series with a load.
- Now know the current coil (CC), it carries the current that is drawn by the load.
- The moving coil of instrument which is wound with thin and light wire for a large number of turns can either be connected across supply or load.
- Now know the pressure coil or PC, it carries a current proportional to voltage.
- A high resistance is connected in series with pressure coil ckt, in order to protect it from large current.



As there is a high resistance connected in p.c. ckt, the p.c. is assumed to be purely resistive.

Drawing the phasor relationship, we get



From the expression for the T_d of an electro-dynamo instrument for sinusoidal currents,

$$T_d = I_1 I_2 \cos \angle_{I_1 I_2} \cdot \frac{dM}{d\theta}$$

$$I_1 = I_{cc} = I$$

$$I_2 = I_{pc} = \frac{V}{R_p} \quad (R_p = R + \delta p)$$

Since, $\angle_{I_1 I_2} = \angle_{I_{cc} I_{pc}} = \phi$

$$\therefore T_d = \frac{VI \cos \phi}{R_p} \cdot \frac{dM}{d\theta}$$

at steady state position, $T_c = T_d$

$$K\theta = \frac{VI \cos \phi}{R_p} \cdot \frac{dM}{d\theta}$$

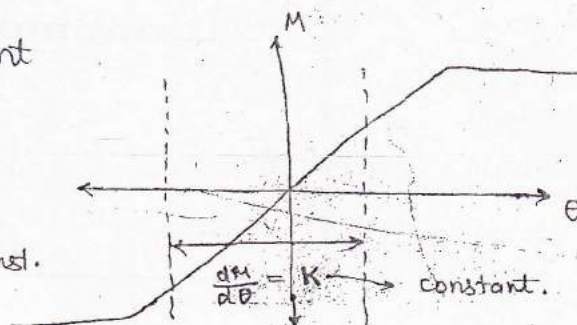
A plot b/w θ and M is drawn;

If the span of the instrument is chosen where θ varies

linearly wot M , then we

have K , $\frac{dM}{d\theta}$ and R_p are const.

$$\therefore \theta \propto VI \cos \phi$$



Note :-

- (i) The ~~pract~~ pressure coil for a practical Electro-dynamo wattmeter is a highly resistive circuit.
- (ii) The electro-dynamo meter measure Average value of the Active power.
- (iii) As $\left(\frac{dM}{d\theta}\right)$ is made constant, $\theta \propto VI \cos\phi$. Hence the scale of a electro-dynamo wattmeter is uniform.

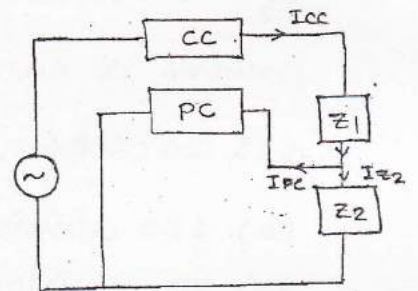
* Q.1 :- P - 8.

$$\begin{aligned}
 P &= VI \\
 &= I \cdot (V_{cc} + V_a + V) \\
 &= 5 \cdot (5 \times 0.2 + 5 \times 0.2 + 200) \\
 P &= 1010 \text{ watt (Ans) (option-d)}
 \end{aligned}$$

across current coil.
 across ammeter.
 across load

Q) A wattmeter is connected as shown in the figure, the wattmeter reads :

- (a) 0 always
 (b) Total power consumed by Z_1 and Z_2 .
 (c) Power consumed by Z_1
 (d) Power consumed by Z_2 .



Ans \Rightarrow As the pressure coil connected across Z_2 . The wattmeter reads power consumed by Z_2 only. (option-d).

Q) The current coil of a 200 V, 5 A electro-dynamo meter type low LPF wattmeter carries a current of $\sqrt{2} \cos(100\pi t)$ A. The voltage across the pressure coil is $\sqrt{2} \sin(100\pi t)$ volt. The meter will indicate :

- (a) 0 watt
 (b) 100 watt
 (c) 200 watt.
 (d) 400 watt.

Ans $\Rightarrow \phi = 90^\circ$ from the expression and $P = VI \cos\phi$. Since $\cos\phi = 0$ The wattmeter indicate zero watt (option-a)

Q) The current through a current coil of a wattmeter is given by, $I = 1 + 2\sin\omega t$ Amp. and the voltage across the pressure coil is $V = 2 + 3\sin 2\omega t$ volts, the wattmeter will reads :-

- (a) 8 watt
(b) 5 watt

- (c) 2 watt
(d) 1 watt

Ans \Rightarrow since phase

\hookrightarrow Note:- Power dissipation in component of dissimilar frequencies will be zero. Hence the watt meter reads only the DC component of power.

Hence $P = 1 \times 2 = 2$ watt (option - c)

Q) A 5A, 110V electro-dynamo wattmeter has a scale having 110 division and its pressure coil is fed by a voltage of $110\sqrt{2} \cos(314t) + \sqrt{2} \sin(628t)$ and its current coil

carries a current of $5\sqrt{2} \cos(314t + 60^\circ) + 2\sqrt{2} \sin(628t + 90^\circ) + \sqrt{2} \cos(642t + 90^\circ)$ Amp, the needle of wattmeter will move to :-

- (a) 110 divisions.
(b) 50 divisions.

- (c) 54 divisions.
(d) 55 divisions.

Ans \Rightarrow The full scale power = $\frac{110\sqrt{2}}{\sqrt{2}} \times \frac{5\sqrt{2}}{\sqrt{2}} \times \cos 0 = 550$.
(when $\phi = 0$)

The power indicated by wattmeter = $\frac{110\sqrt{2}}{\sqrt{2}} \times \frac{5\sqrt{2}}{\sqrt{2}} \times \cos 60^\circ$
= 275 watts.

Now, $550 \text{ W} \longrightarrow 110 \text{ division}$
 $275 \text{ W} \longrightarrow ? \text{ division}$

= $\frac{275 \times 110}{550} = 55 \text{ division}$

(option - d)

Errors in Electro-dynamo wattmeter :-

(1) Errors due to pressure coil Inductance :-

- As a pressure coil of E.D. wattmeter is highly resistive ckt, it is associated with a small but finite value of an inductance, which introduces an error in reading.

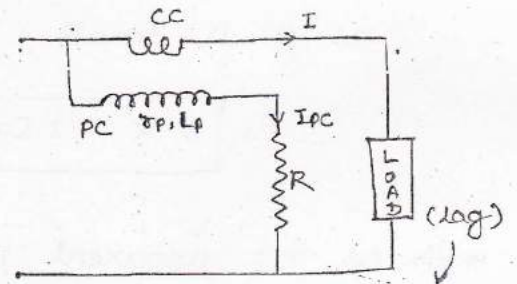
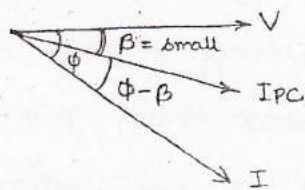
* Effect on the pressure coil inductance :-

(a) Effect on the pressure coil inductance on the reading of the wattmeter :-

↳ The effect of this finite but small pressure coil inductance for leading as well as lagging load as discuss below :-

(i) Lagging Load :-

• Drawing the phasor relation by taking the inductance of PC into consideration.



(Generally in N/w Load is lagging in nature so we consider in our ckt to get expression)

From the expression for T_d of an ED Instrument for sinusoidal current we have,

$$T_d = I_1 I_2 \cos \theta \frac{dM}{d\theta}$$

$$I_1 = I_{pc} = I$$

$$I_2 = I_{pc} = \frac{V}{Z_p}$$

$$Z_p = R + r_p + j\omega L_p$$

$$Z_p = R_p + j\omega L_p$$

$$[\because R_p = R + r_p]$$

$$Z_p = \sqrt{R_p^2 + (\omega L_p)^2}$$

$$\beta = \tan^{-1} \left(\frac{\omega L_p}{R_p} \right)$$

Since, $\angle I_1 I_2 = \angle I_{cc} I_{pc} = (\phi - \beta)$

$$T_d = \frac{VI}{Z_p} \cos(\phi - \beta) \cdot \frac{dM}{d\theta}$$

$$\cos \beta = \frac{R_p}{Z_p}$$

$$\therefore T_d = \frac{VI}{R_p} \cos \beta \cdot \cos(\phi - \beta) \cdot \frac{dM}{d\theta}$$

at steady state position,

$$\theta \propto VI \cos \beta \cdot \cos(\phi - \beta)$$

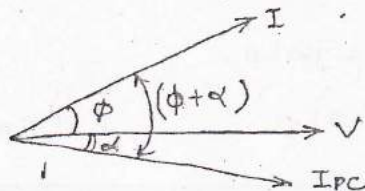
as ' β ' is small, $\cos \beta \approx 1$.

$$\therefore \theta \propto VI \cos(\phi - \beta)$$

* ii) As the apparent power factor $(\phi - \beta)$ is less than the true power factor angle ' ϕ ', resulting in the apparent power factor $\cos(\phi - \beta)$ being greater than the actual power factor $\cos \phi$. Hence the reading of wattmeter will be higher than the actual power consumed by the load due to the effect of pressure coil inductance on lagging load.

(ii) Leading Load :-

Drawing the phasor relationship by taking the inductance of pressure coil into consideration.



If we repeat the same, we get.

$$\theta \propto VI \cos(\phi + \alpha)$$

- As the apparent power factor angle $(\phi + \alpha)$ is greater than the true power factor angle (ϕ) resulting in the apparent power factor $\cos(\phi + \alpha)$ being less than the true power factor $\cos\phi$. Hence the reading of the wattmeter will be lower than the actual power consumed by the load due to effect of PC inductance on leading load.

(b) Magnitude of the error due to PC inductance :- → (Pressure coil)

- The correction factor of an instrument is the ratio of its true value and the measured value.

In this case,

$$\text{correction factor, } \left\{ \text{c.f.} = \frac{TP}{MP} \right\}$$

where, TP = True value of power.

MP = wattmeter reading (Measured value of power)

$$\begin{aligned} \therefore \text{C.F.} &= \frac{VI \cos\phi}{VI \cos\beta \cdot \cos(\phi - \beta)} \\ &= \frac{\cos\phi}{\cos\beta (\cos\phi \cos\beta + \sin\phi \sin\beta)} \\ &= \frac{\cos\phi}{\cos^2\beta (\cos\phi \cos\beta + \sin\phi \sin\beta)} = \frac{\cos\phi}{\cos^2\beta (\cos\phi + \sin\phi \cdot \tan\beta)} \end{aligned}$$

As β is small, $\cos^2\beta \approx 1$.

$$= \frac{\cos\phi}{\cos\phi (1 + \tan\phi \cdot \tan\beta)}$$

$$\boxed{C.F. = \frac{1}{1 + \tan\phi \cdot \tan\beta}} \quad \dots (1)$$

From the definition of correction factor is :-

$$C.F. = \frac{TP}{MP} = \frac{1}{1 + \tan\phi \cdot \tan\beta}$$

$$MP = TP + \tan\phi \cdot \tan\beta * TP$$

$$MP - TP = \tan\phi \cdot \tan\beta * TP$$

$$\therefore \boxed{\text{Error} = \tan\phi \cdot \tan\beta * (\text{True Power})} \quad \dots (2)$$

$$\% \text{ Error} = \left(\frac{MV - TV}{TV} \right) \times 100$$

In this case,

$$\% \text{ Error} = \frac{\tan\phi \cdot \tan\beta * (\text{True Power})}{(\text{True power})} \times 100$$

$$\boxed{\% \text{ Error} = \tan\phi \cdot \tan\beta * 100} \quad \dots (3)$$

* Note :- From expression (2) it can be seen that the magnitude of the coil due to pressure coil inductance is higher at low power factor loads.

(c) Compensation due to presence of PC inductance :-

- The fact that PC inductance could introduce error in the measurement become evident while representing the PC current, I_{pc} where $\left(I_{pc} = \frac{V}{Z_p} \right)$
- Thus if Z_p is make equal to R_p i.e. the PC make purely

resistive, then these errors can be compensated.

- This is done by introducing a proportionate reactive capacitance in the pressure coil circuit :-

(a) A capacitance in series with a pressure coil ckt :-

Calculate the Impedance of the PC we have

$$Z_p = r_p + j\omega L_p + R + \frac{1}{j\omega C}$$

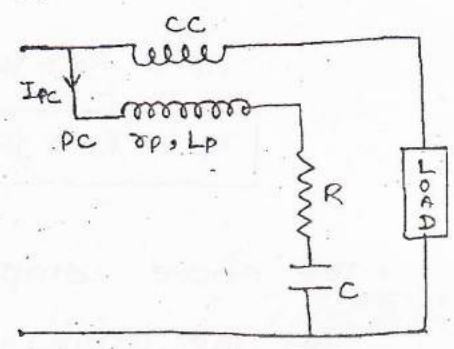
$$Z_p = R_p + j\omega L_p + \frac{1}{j\omega C} \quad [R_p = r_p + R]$$

$$Z_p = R_p + j \left(\omega L_p - \frac{1}{\omega C} \right)$$

If the value of 'C' is so chosen that,

$$\left\{ \omega L_p - \frac{1}{\omega C} = 0 \right\}$$

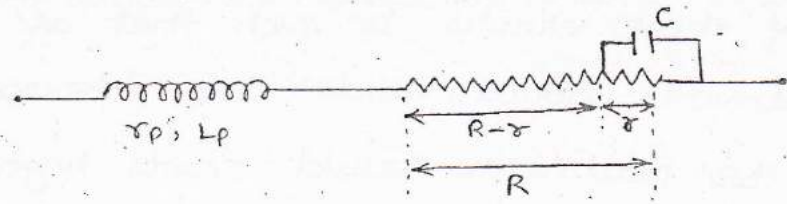
then, $\{ Z_p = R_p \}$.



* Note :- As $\left(\omega L_p - \frac{1}{\omega C} = 0 \right)$ is a frequency dependent expression

This compensation methodology is never used.

(b) A capacitance across certain section of the high resistance placed in series with the pressure coil :-



Calculating the impedance of PC we have,

$$Z_p = r_p + j\omega L_p + (R - \delta) + \delta \parallel \frac{1}{j\omega C}$$

$$Z_p = R_p + j\omega L_p - \delta + \frac{\delta}{1 + j\omega C \delta} \quad [\because R_p = R + r_p]$$

Multiplying and dividing by $(1 - j\omega C\delta)$ with $\frac{\delta}{1 + j\omega C\delta}$

$$Z_p = R_p + j\omega L_p - \delta + \frac{\delta(1 - j\omega C\delta)}{1 + \omega^2 C^2 \delta^2}$$

If ' δ ' is chosen to be small then,

$$1 + \omega^2 C^2 \delta^2 \approx 1$$

$$Z_p = R_p + j\omega L_p - \delta + \delta - j\omega C\delta^2$$

$$Z_p = R_p + j\omega (L_p - C\delta^2)$$

- The above compensation methodology is fails ^{in instances} where :-
 - The value of ' δ ' increases.
 - The frequency increases beyond 10 KHz.

↳ Note :- The above compensation methodology also compensate for the error due to frequency which occur because of the variation in frequency.

(2) Error due to pressure coil Capacitance :-

- The capacitive reactance of the pressure coil ckt is due to the stray or distributed capacitances of the pressure coil.
- Effect of these errors is such that at lagging loads, the wattmeter would reads low whereas at leading loads, the wattmeter would reads high.
- If the inductive and capacitive reactance of the PC ckt are made equal at the design stage, then both the PC inductance and PC capacitance errors are compensated.

Q) A electrodynamic wattmeter measures power in a single phase ckt. Load voltage 230V and current 5A at a lagging power factor of 0.1. The pressure coil resistance is 10K Ω and its inductance is 100 mH. Calculate the % error in the instrument if the supply frequency of 50 Hz.

Ans $V = 230 \text{ V}$; $I = 5 \text{ A}$

$$\cos \phi = 0.1 \Rightarrow \phi = 84.26^\circ$$

$$\tan \phi = 9.94$$

$$R_p = 10 \text{ K}\Omega \text{ , } L_p = 100 \text{ mH} \text{ , } f = 50 \text{ Hz}$$

$$\% \text{ Error} = \tan \phi \cdot \tan \beta \times 100$$

$$\therefore \tan \beta = \frac{\omega L_p}{R_p} = \frac{2\pi \times 50 \times 100 \times 10^{-3}}{10 \times 10^3} = 0.00314$$

$$\text{So, } \% \text{ Error} = 9.94 \times 0.00314 \times 100$$

$$\% \text{ Error} = \approx 3.1 \% \quad (\text{Ans})$$

Q) The inductive reactance of PC ckt of ED wattmeter is 0.4% of its resistance at Normal frequency (50 Hz) and its capacitance is negligible. Calculate the % error and the correction factor due to the reactance for a load at 0.707 lagging power factor.

Ans $\cos \phi = 0.707$

$$\phi = 45^\circ \text{ , } \tan \phi = 1$$

$$\therefore \frac{\omega L_p}{R_p} = 0.4\%$$

$$\tan \beta = \frac{0.4}{100} = 0.004$$

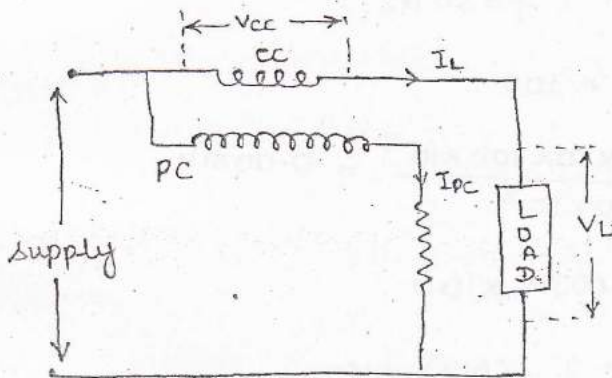
$$\therefore \text{C.F.} = \frac{1}{1 + \tan \phi \cdot \tan \beta}$$

$$= \frac{1}{1 + 1 \times 0.004} = 0.996$$

$$\begin{aligned} \% \text{ Error} &= \tan\phi \cdot \tan\beta \times 100 \\ &= 1 \times 0.004 \times 100 \\ &= 0.4 \% \quad (\text{Ans}) \end{aligned}$$

(3) Error due to pressure coil connection :-

- This error occurs due to side on which the pressure coil is connected. The two connection methodology of PC are shown in the figures below :-

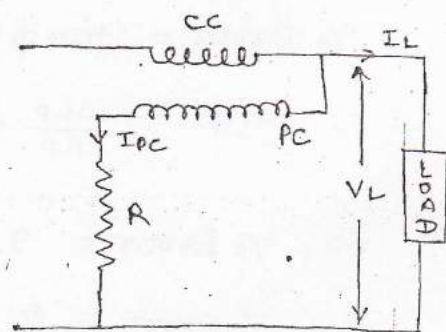


(Fig-1)

wattmeter reading,

$$\begin{aligned} &= I_L (V_{cc} + V_L \cos\phi) \\ &= I_L [I_L \cdot r + V_L \cos\phi] \\ &= I_L^2 r + V_L I_L \cos\phi. \end{aligned}$$

$$\% \text{ Error} = \frac{I^2 r}{V I \cos\phi} \times 100$$



(Fig-2)

wattmeter reading,

$$\begin{aligned} &= V_L (I_{pc} + I_L \cos\phi) \\ &= V_L \left[\frac{V_L}{R} + I_L \cos\phi \right] \\ &= \frac{V_L^2}{R} + V_L I_L \cos\phi. \end{aligned}$$

$$\% \text{ Error} = \frac{V^2/R}{V I \cos\phi} \times 100$$

- In fig-1, where the current coil is connected along the load, the voltage as seen by the pressure coil act is the sum of the voltage drop across the current coil and the load.
- Thus from the above analysis it can be seen that the wattmeter reading in this connection methodology not

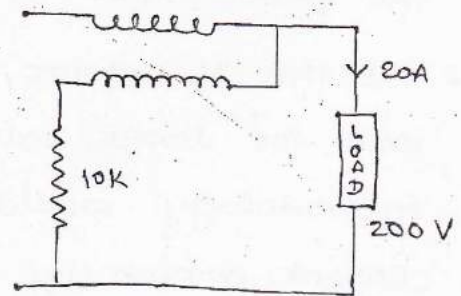
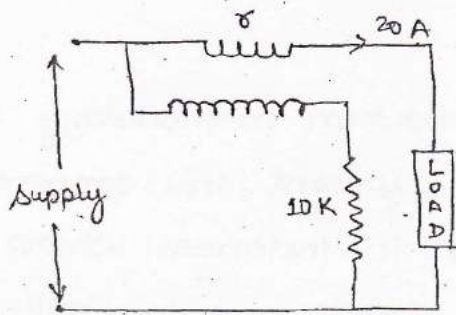
only ~~contains~~ the ~~to~~ contain a component of the power dissipated by the load but also contain a component of the power loss in coil ($I^2 R$).

- As the % error in this connection methodology decreases with for lower values of load current, this connection methodology would be suitable in instances where the current drawn by the load is small and the voltage high / high impedance loads / low power circuits.
- In figure-2, when the pressure coil is connected across the load, the current through the current coil is the sum of the current drawn by the load and the pressure coil.
- Thus, from the above analysis it can be seen that the wattmeter reading in this connection methodology would not only contain a component of power dissipated by the load but would also contain a component of the power loss in the pressure coil (V^2/R).
- As the power loss in the pressure coil ~~is~~ is a small and a constant value (as R is high), the % error in this case decreases in instances where the load current drawn is large and the voltage is small / ~~the~~ low impedance load / high power circuits.

* Note:- A ~~gener~~ general purpose electro-dynamo wattmeter is usually compensated for the error due to power consumed in its pressure coil circuit.

Q) Two types of wattmeter connections are shown the figure. The value of wattmeter current coil resistance which makes the

connection error is same in both cases is :-



Ans Acc. to question,

$$I^2 \delta = \frac{V^2}{R}$$

$$\delta = \frac{V^2}{I^2 R} = \frac{(200)^2}{(20)^2 \times 10 \times 10^3} = 0.01 \Omega \quad (\text{Ans})$$

Q) The ckt in the figure is used to measure the power consumed by the load. The current coil and voltage coil have 0.02Ω and 1000Ω resistances resp. The measured power compared to true power will be :

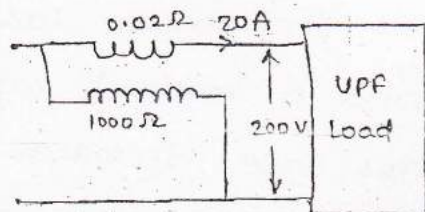
(a) 0.4% less.

(c) 0.2% more.

(b) 0.2% less.

(d) 0.4% more.

$$\begin{aligned} \text{Ans} \Rightarrow \% \text{ Error} &= \frac{I^2 \delta}{VI \cos \phi} \times 100 \\ &= \frac{(20)^2 \times 0.02}{200 \times 20} \times 100 \end{aligned}$$



$$= +0.2\% \quad (\text{option - c})$$

Q) A wattmeter as a current coil of 0.03Ω and a pressure coil of 6000Ω resistance. Calculate the % error if the wattmeter is so connected that :-

(i) The CC is connected to the load side

(ii) The PC is connected to the load side.

If the load takes $20A$ at a voltage of $220V$ and 0.6 power

factor in each case. What load current would give equal error for both the connection.

Ans $\Rightarrow r = 0.03 \Omega$, $R = 6000 \Omega$, $I = 20 \text{ A}$, $V = 220 \text{ V}$, $\cos \phi = 0.6$.

(i) CC connected on load side,

$$\begin{aligned} \% \text{ Error} &= \frac{I^2 r}{VI \cos \phi} \times 100 \\ &= \frac{(20)^2 \times 0.03}{220 \times 20 \times 0.6} \times 100 \\ &= 0.45 \% \end{aligned}$$

(ii) PC connected on load side,

$$\begin{aligned} \% \text{ Error} &= \frac{V^2 / R}{VI \cos \phi} \times 100 \\ &= \frac{(220)^2}{220 \times 20 \times 0.6 \times 6000} \times 100 \\ &= 0.305 \% \end{aligned}$$

(iii) Load current that gives equal error,

$$I^2 r = V^2 / R$$

$$I = \sqrt{\frac{V^2}{R \times r}} = \sqrt{\frac{(220)^2}{6000 \times 0.03}}$$

$$I = 16.39 \text{ Amperes.}$$

Q) An E.D. wattmeter is applied to measure power of single phase a.c. The load voltage is 230 V and load current is 5 A at a lagging power factor of 0.1. The wattmeter potential coil has a resistance of 10,000 Ω and a inductive reactance negligible compare to its resistance. Determine the % error in the wattmeter reading if the inductance of coil is 100 mH and pressure coil is connected on load side.

Ans → voltage = 230 V, current = 5 A, $\cos \phi = 0.1$

$$L_p = 100 \text{ mH}, R_p = 10,000 \Omega$$

Assume $f = 50 \text{ Hz}$.

$$\text{Here, \% Error} = \frac{MP - TP}{TP} \times 100$$

Actual value of power consumed,

$$\begin{aligned} TP &= VI \cos \phi \\ &= 230 \times 5 \times 0.1 \\ &= 115 \text{ watts.} \end{aligned}$$

wattmeter's reading,

$$= TP + \frac{V^2}{R} + \tan \phi \cdot \tan \beta * TP$$

$$\therefore \text{Power loss in } PC = \frac{V^2}{R}$$

$$= \frac{230 \times 230}{10,000} = 5.29 \text{ watt}$$

Now, error due to pressure coil inductance,

$$= \tan \phi \tan \beta * TP = 9.94 \times 3.14 \times 10^{-3} * 115 = 3.59 \text{ W}$$

$$\therefore \cos \phi = 0.1 \Rightarrow \phi = 84.26^\circ$$

$$\tan \phi = 9.94$$

$$\therefore \tan \beta = \frac{\omega L_p}{R_p} = \frac{2 \times \pi \times 100 \times 10^{-3}}{10,000} = 3.14 \times 10^{-3}$$

Now,

$$MP - TP = \frac{V^2}{R} + \tan \phi \tan \beta * TP$$

$$= 5.29 + 3.59$$

$$= 8.88 \text{ watts}$$

$$\therefore \% \text{ Error} = \frac{8.88}{115} \times 100$$

$$= 7.72 \% \quad (\text{Ans})$$

Q) The resistances of two coils of a wattmeter are 0.02Ω and 100Ω respectively and both are non-inductive, the current through a resistive load is 20 A and voltage across it, is 30 V . In one of two ways of connecting the voltage coil, the error in the reading would be:

- (a) 0.1% too high
 (b) 0.2% too high
 (c) 0.15% too high.
 (d) zero.

Ans: Since $I^2 r = (20)^2 \times 0 = 0$
 So, error = 0 (option-d)

(4) Error due to Eddy current :-

- The eddy currents are induced in all the conductors that are present in the vicinity (near) of the fixed coil flux. Thereby reducing the number of active flux interaction resulting in a lower sensitivity.
- These errors are compensated by properly insulating the conductor present in the vicinity of the fixed coil.

(5) Error due to stray magnetic field :-

- This error is compensated by providing proper magnetic shielding.

Q) Low power factor Electro-dynamo meter wattmeter :-

- A Normal Electro-dynamo type of wattmeter could be unsuitable for measuring power in a low power factor ckt as :-

(i) As the value of power factor tends to zero, the angular deflection of the instrument becomes negligible as $\theta \propto \cos \phi$.

(ii) The error due to PC inductance in a low power factor ckt are large as $\phi \rightarrow 90^\circ$.

In order to make electro-dynamo wattmeter suitable to measure power in low power factor ckt; the following modifications are made:-

⇒ Reason-1:-

as, ~~$\cos \phi$~~ $\phi \rightarrow 90^\circ$

$\cos \phi \rightarrow 0$

$\therefore T_d \propto V I \cos \phi$

$\therefore T_d \rightarrow 0$

Hence, $\theta \rightarrow 0$.

↳ Solution:-

(a) $R_p \downarrow \downarrow \downarrow$

$\Delta \phi$, $I_{pc} \uparrow \uparrow \uparrow$

\therefore as $T_d \propto I_{pc}$

$T_d \uparrow \uparrow \uparrow$

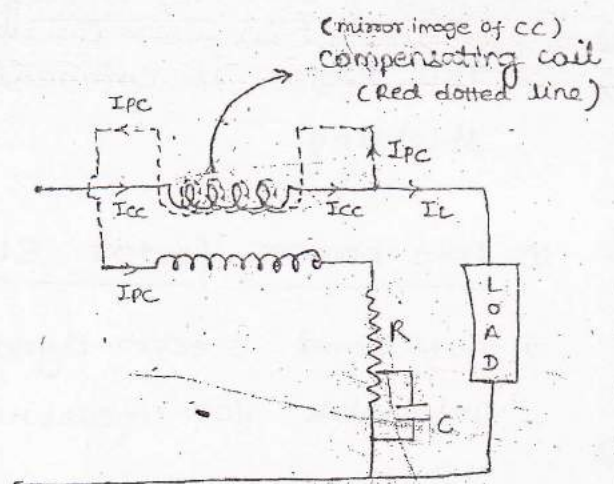
(b) $I_{cc} = I_L + I_{pc}$

$\phi_{cc} = \phi_L + \phi'_{pc}$

$\phi_{comp} = -\phi'_{pc}$

Net flux = ϕ_L
due to CC

↘ interact with ϕ_{pc}



(c) $T_c \uparrow \uparrow \uparrow$

So, $\theta \uparrow \uparrow \uparrow$

⇒ Reason-2 :-

as $\phi \uparrow \uparrow \uparrow$

$\tan \phi \uparrow \uparrow \uparrow$

∴ $\tan \phi \tan \beta \neq TP \uparrow \uparrow \uparrow$

→ Solution :- Connecting suitable capacitance across high resistance in series with PC ckt.

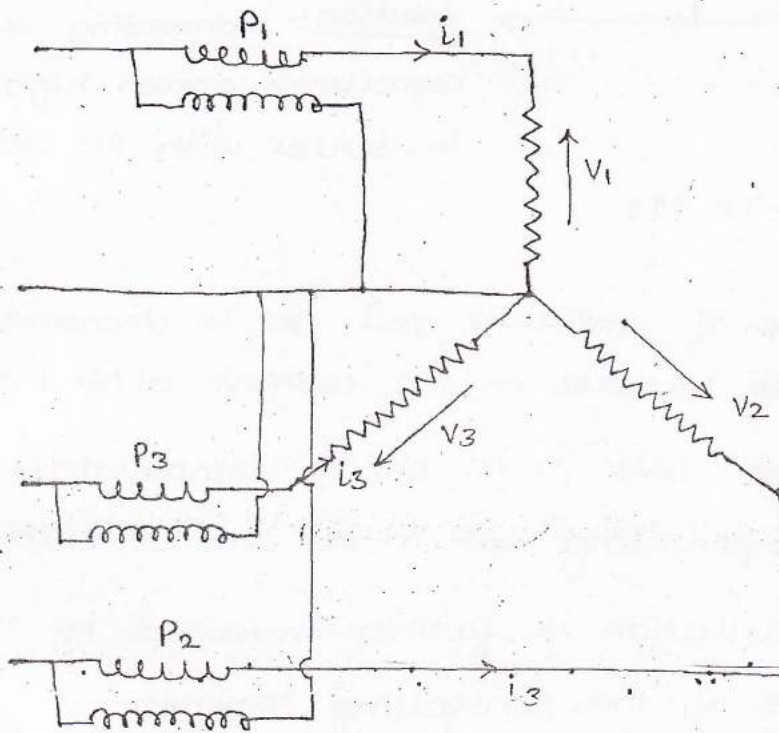
- (a) The resistance of pressure coil ckt is decreased, resulting in an increase in PC current which increases T_d .
- (b) The high power loss in PC ckt is compensated by winding a compensating coil in the current coil circuit.
- (c) The angular deflection is further increased by reducing the magnitude of the controlling Torque.
- (d) The increased error due to PC inductance are compensated by connecting a suitable capacitance across the high resistance connected in series with PC ckt.

⊙ Poly-Phase power measurement :-

- Measurement of power in poly-phase ckt using single phase electro-dynamo type of wattmeter are governed by the Blondel's theorem which states :-

(i) In a n -phase $(n+1)$ -wire system supplying either a balanced or unbalanced load n -wattmeters are required for the measurement of the power

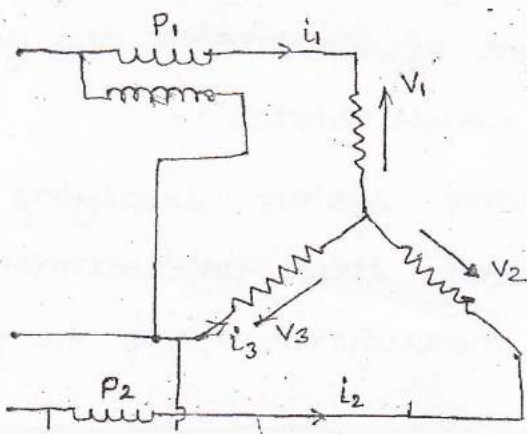
where the current coil of n -wattmeters are connected in the respective phases and their pressure coil are connected between that phase and the common line.



Power

$$\begin{cases} P = i_1 V_1 + i_2 V_2 + i_3 V_3 \\ = P_1 + P_2 + P_3 \end{cases}$$

(ii) In a n -phase n -wire system supplying either a balanced or unbalanced load $(n-1)$ -wattmeters are required for the measurement of total power where the current coils of $(n-1)$ wattmeters are connected in the respective phases and their pressure coils are connected b/w that phase and the phase is designated as common phase.



Power, $P = P_1 + P_2$

$$\begin{aligned} &= i_1 (V_1 - V_3) + i_2 (V_2 - V_3) \\ &= i_1 V_1 - i_1 V_3 + i_2 V_2 - i_2 V_3 \\ &= i_1 V_1 + i_2 V_2 - V_3 (i_1 + i_2) \end{aligned}$$

Apply KCL,

$$i_1 + i_2 + i_3 = 0$$

$$i_3 = -(i_1 + i_2)$$

$$0 = i_1 V_1 + i_2 V_2 + i_3 V_3$$

* Effect of the power factor on reading of the wattmeter

(i) $\phi = 0$, $\cos \phi = 1$, $P_1 = \frac{P}{2}$ and $P_2 = \frac{P}{2}$.

therefore, $P_1 + P_2 = P$.

Hence both the watt-meters indicate half of the power consumed by the load.

(ii) $\phi = 60^\circ$, $\cos \phi = 0.5$

one of the watt-meters indicate zero while the other indicate the total power consumed by the load.

(iii) $\phi = 90^\circ$, $\cos \phi = 0$, $P_1 = P/2$, $P_2 = P/2$

But one of the wattmeter indicates negative reading

Therefore, $P_1 + P_2 = 0$.

↳ Negative reading ⁱⁿ of the wattmeters indicate is obtained by reversing the connection of current coil.

(iv) $0^\circ < \phi < 59^\circ$.

Both wattmeters will gives a positive reading.

(v) $61^\circ < \phi \leq 90^\circ$.

One of the wattmeter will always indicate a negative value.

* The reading of the wattmeter can be used to find the power factor angle,

$$\left[\phi = \tan^{-1} \left\{ \frac{\sqrt{3} (P_1 - P_2)}{P_1 + P_2} \right\} \right]$$

Q) The power of a 3-phase, 3-wire balanced system was measured by 2-wattmeter method, the reading one of

the wattmeter is found to be doubled that of the other. What is power factor..

(a) 1

(c) 0.707

(b) 0.866

(d) 0.5

$$\text{Ans} \Rightarrow \phi = \tan^{-1} \left(\frac{\sqrt{3} (2P - P)}{2P + P} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{3} P}{3P} \right)$$

$$= \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ$$

$$\therefore \cos \phi = \text{Power factor} = \cos 30^\circ = 0.866 \quad (\text{Option-b})$$

Q) Two wattmeters which are connected to measure the total power in a 3-phase 3-wire system supplying a balanced load read -3.5 kW and $+8 \text{ kW}$ respectively.

The power factor and input power are :-

(a) 0.2655, 4.5 W

(c) 0.829, 11.5 W

(b) 0.2655, 11.5 W

(d) 0.829, 4.5 W

Ans \Rightarrow one reading is -ve, so power factor is less than 0.5, so, option (c) and (d) becomes wrong.

$$\text{Now, } P_{in} = P_1 + P_2$$

$$= -3.5 + 8 = 4.5 \quad (\text{option-a})$$

① Measurement of Resistance :-

\Rightarrow Topics :-

1. Introduction,

2. Measurement of medium Resistance (R)

(a) Wheatstone Bridge.

- (b) Substitution method .
- (c) Voltmeter Ammeter method .
- (d) Ohm meter method .

- 3. Measurement of low Resistance (R)
- 4. Measurement of High Resistance (R)

⇒ Introduction :-

- Resistance measurement characterised error, due to magnitude of resistance being measured. Thus resistance are classified on the basis of their magnitude as:-
 - (1) Low Resistance ($R < 1 \Omega$)
 - (2) Medium Resistance ($1 < R < 100 K\Omega$)
 - (3) High Resistance ($R > 100 K\Omega$)

⇒ Measurement of Medium Resistance :-

(1) Wheatstone Bridge method :-

"This bridge measure the value of an unknown resistance in terms of a standard resistance."

→ at balance ,

$$I_D = 0$$

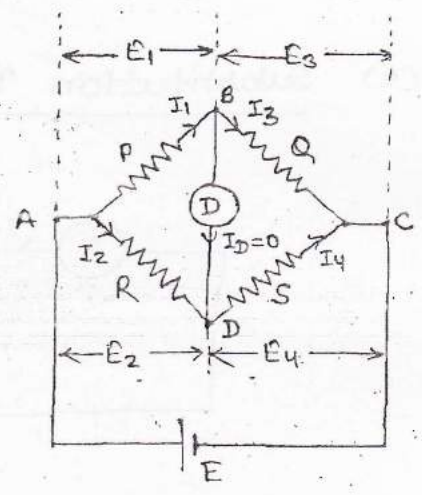
$$V_B = V_D$$

$$E_1 = E_2$$

and $E_3 = E_4$

$$I_1 P = I_2 R \dots (1)$$

Here, $I_1 = I_3 = \frac{E}{P+Q}$



$$I_2 = I_4 = \frac{E}{R+S}$$

Substitute these value in eqⁿ (1),

$$\frac{EP}{P+Q} = \frac{ER}{R+S}$$

$$R(P+Q) = P(R+S)$$

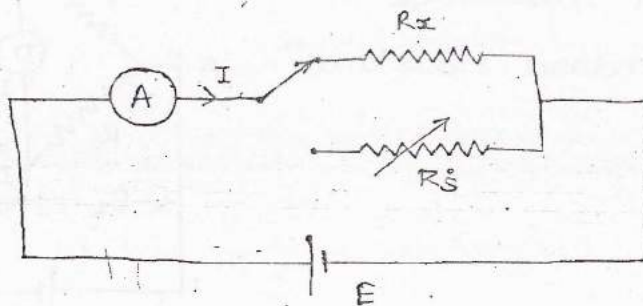
$$RP + RQ = PR + PS$$

$$RQ = PS$$

$$\therefore \boxed{R = \frac{P \cdot S}{Q}}$$

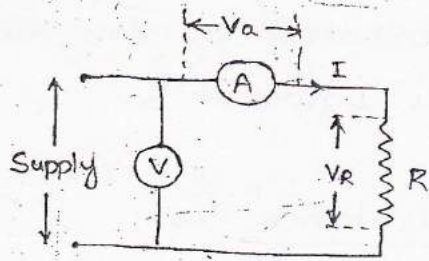
From above expression, it can be said that, the wheatstone bridge is the most accurate method for the measurement of medium resistances as it basis of operation on the principle of comparison and null deflection.

(2) Substitution Method :-

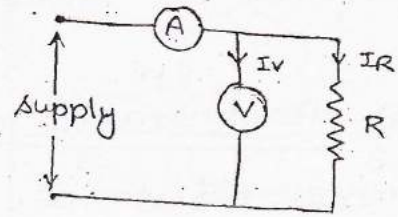


- The substitution method for the measurement of medium resistance, measure the value of unknown resistance by comparing it with standard resistance in terms of the current drawn by the standard and the unknown. This methodology gives sufficiently high

accuracy as it basis its operation on the principle of comparison.



(Fig-1)



(Fig-2)

$$\begin{aligned}
 R_{mv} &= \frac{\text{voltmeter Reading}}{\text{Ammeter Reading}} \\
 &= \frac{V_a + V_R}{I} \\
 &= \frac{I R_a + I R}{I}
 \end{aligned}$$

$$R_{mv} = R_a + R$$

$$\left\{ \% \text{ Error} = \frac{R_a}{R} \times 100 \right\}$$

$$\begin{aligned}
 R_{mv} &= \frac{\text{voltmeter Reading}}{\text{Ammeter Reading}} \\
 &= \frac{V}{I_v + I_R} \\
 &= \frac{V}{\frac{V}{R_v} + \frac{V}{R}} \\
 &= \frac{1}{\frac{1}{R_v} + \frac{1}{R}}
 \end{aligned}$$

$$R_{mv} = \frac{R}{1 + R/R_v}$$

In Fig-1, as the % Error decreases for larger value of the resistance under measurement, this connection methodology is suitable for measurement of larger value of medium resistance (R)

In Fig-2, it can be seen that measured value tends to be true value for smaller value of resistance under measurement, this connection methodology is suitable for lower value of medium resistance (R)

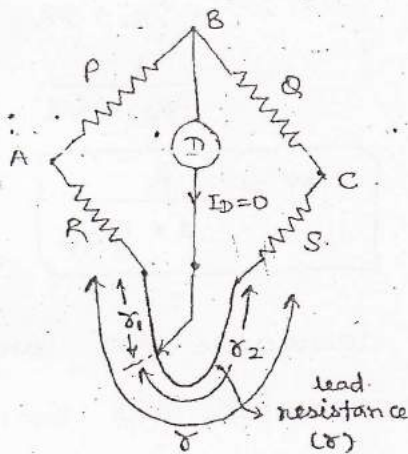
Note:- The voltmeter - Ammeter method where voltmeter connected across the supply can also be used for the

measurement of low resistance ($R < 1\Omega$)

- Medium resistance are fabricated by two terminal devices as shown in below figure.

(3) Measurement of Low Resistance :-

- Low resistance measurement is characterised by the error due to the effect of lead resistance.
- The effect of this lead resistance on the measurement of unknown resistance R can be understood from the analysis given below.



→ Ideally,

$$R = \frac{P}{Q} S \quad \dots (1)$$

In this case,

$$(R + r_1) = \frac{P}{Q} (S + r_2) \quad \dots (2)$$

$$R = \frac{P}{Q} S + \left(\frac{P}{Q} r_2 - r_1 \right)$$

- The effect of this lead resistance, can be eliminated by introducing a extra set of ratio arm in the wheat stone bridge circuit under specific condition to form the "Kelvin double bridge circuit."

⇒ Kelvin Double Bridge ckt :-

Here,

$$\frac{P}{Q} = \frac{r_1}{r_2} \quad \dots (3)$$

but $\sigma_1 + \sigma_2 = \sigma$ ----- (4)

Adding 1 on both side of eqⁿ (3) we have,

$$\frac{P}{Q} + 1 = \frac{\sigma_1}{\sigma_2} + 1$$

$$\frac{P+Q}{Q} = \frac{\sigma_1 + \sigma_2}{\sigma_2}$$

As $\sigma_1 + \sigma_2 = \sigma$

$$\frac{P+Q}{Q} = \frac{\sigma}{\sigma_2}$$

$$\sigma_2 = \frac{Q\sigma}{P+Q}$$
 ----- (5)

Taking the reciprocal of (3) and adding (1) on both side we have, ...

$$\frac{Q}{P} + 1 = \frac{\sigma_2}{\sigma_1} + 1$$

$$\frac{Q+P}{P} = \frac{\sigma_2 + \sigma_1}{\sigma_1}$$

As $\sigma_1 + \sigma_2 = \sigma$

$$\sigma_1 = \frac{P\sigma}{P+Q}$$
 ----- (6)

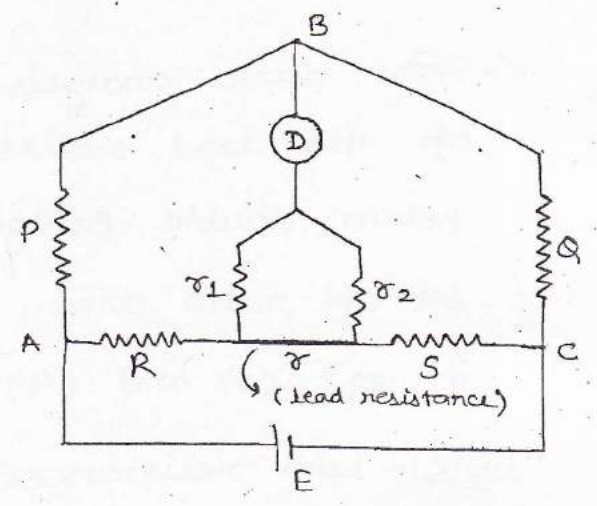
substituting (5) and (6) in (2), we have

$$R + \frac{P\sigma}{P+Q} = \frac{P}{Q} \left\{ S + \frac{Q\sigma}{P+Q} \right\}$$

$$R + \frac{P\sigma}{P+Q} = \frac{PS}{Q} + \frac{PQ\sigma}{Q(P+Q)}$$

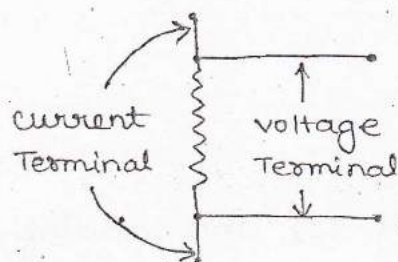
$R = \frac{P}{Q} \cdot S$

----- (7)



↳ From above analysis it can be seen that the effect of the lead resistance can be eliminated in a Kelvin double Bridge ckt by introducing an extra set of ratio arm, under the condition specifying in eqⁿ (3) and (4).

Note:- Low resistances are generally fabricated as four terminal devices as shown below :-

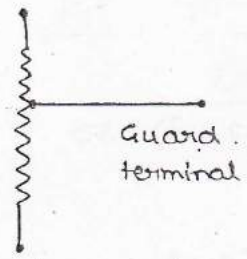


(4) High Resistance Measurement :-

- High Resistance measurement is characterized by the errors due to the leakage currents.
- The various method for the measurement of high resistance are :-
 - (a) Meg-ohm bridge method.
 - (b) Price-guard wire bridge method.
 - (c) Loss of charge method.
 - (d) Insulation testing Megger.

Note :- The Insulation Testing Megger can also be used for measuring earth Resistance.

* High resistances are fabricated as three terminal devices as shown below :-

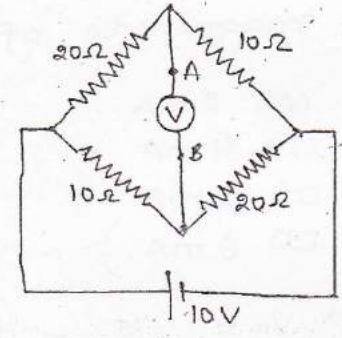


Q) The reading of the high impedance voltmeter shown in the figure below is :

Ans →
$$V_{AB} = 10 \left\{ \frac{20}{10+20} - \frac{10}{10+20} \right\}$$

$= 3.33 \text{ V.}$

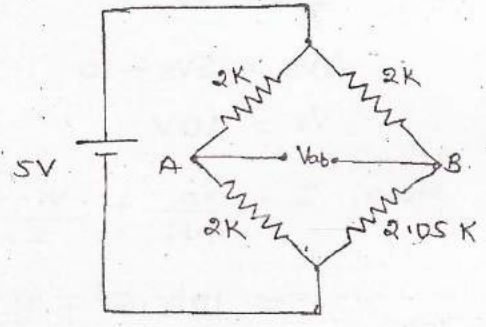
(Ans)



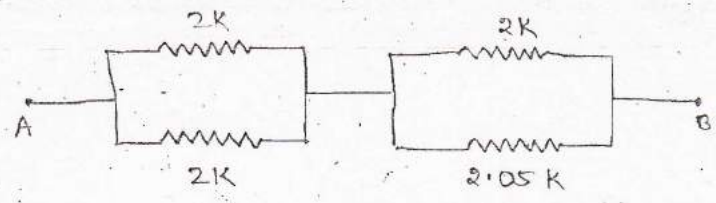
Q) A wheatstone bridge has resistances as shown in the figure below :- If a Galvanometer is having a internal resistance of 50Ω is used for null deflection. Find the value of the off-set current for the resistance shown in the figure.

Ans →
$$V_{ab} = 5 \left\{ \frac{2.05}{2+2.05} - \frac{2}{2+2} \right\}$$

$= 0.03 \text{ V}$



Now,



From the above figure,

$$R_{TH} = \frac{2 \times 2}{2+2} + \frac{2 \times 2.05}{2+2.05}$$

$$= 2.012 \text{ K}\Omega$$